Journal of Economic Dynamics & Control 37 (2013) 2710–2728

Contents lists available at ScienceDirect

Journal of Economic Dynamics & Control

journal homepage: www.elsevier.com/locate/jedc

Design limits and dynamic policy analysis

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ARTICLE INFO

Article history:
Received 23 February 2012
Accepted 7 July 2013
Available online 26 July 2013

JEL classification:
C52
E6

Keywords:
Design limits
Stabilization policy
Frequency domain
Monetary policy rules

ABSTRACT

This paper characterizes the frequency domain properties of feedback control rules in linear systems in order to better understand how different policies affect outcomes frequency by frequency. We are especially concerned in understanding how reductions of variance at some frequencies induce increases in variance at others. Tradeoffs of this type are known in the control literature as design limits. Design limits are important in understanding the full range of effects of macroeconomic stabilization policies. We extend existing results to account for discrete time bivariate systems with rational expectations. Application is made to the evaluation of monetary policy rules.

1. Introduction

This paper explores a set of constraints on the effects of control policies on fluctuations from the perspective of the frequency domain. Aspects of these constraints were initially discussed in Brock and Durlauf (2004, 2005) and Brock et al. (2008a) but otherwise do not appear to have been previously explored in economics contexts. The constraints we study represent fundamental limits on the effects of alternative policies in the sense that they describe how frequency-specific tradeoffs in volatility generically apply to linear feedback rules.

Frequency-specific fluctuations represent an aspect of the effects of policies that has received little attention from economist. Suppose one is considering how different controls affect the stochastic process of a state variable x_t. Underlying the statistic $\text{var}(x_t|C)$, the variance of the process given a control C is the spectral density of x given the control, $f_{x|C}(\omega)$, because the variance is the integral of the spectral density, i.e.

$$\text{var}(x_t|C) = \int_{-\pi}^{\pi} f_{x|C}(\omega)d\omega \tag{1}$$

In fact, the spectral representation of the variance of the state means one can understand the state variance as the sum of the variances from random and orthogonal sines and cosines of different frequencies. By implication, calculations of the effects of a rule on the overall variance mask the effects on fluctuations at the different frequencies in $[-\pi,\pi]$. Further, Eq. (1) hints at the idea that a rule that minimizes the overall variance may exacerbate fluctuations at certain frequencies. A major

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0165-1889/$ - see front matter
http://dx.doi.org/10.1016/j.jedc.2013.07.008

goal of this paper is to determine under what circumstances this must happen and what forms such fundamental tradeoffs take. In the control literature, these tradeoffs are known as *design limits*.

Design limits are a well established area of study in control theory.[1] An important class of results of this type is sometimes known as Bode integral constraints, after Hendrik Bode who first proposed them in 1930s. One methodological contribution of this paper is that we derive frequency tradeoffs for multiple-input multiple-output (MIMO) systems for both forward and backward looking systems. Some of our discrete time results for backward-looking systems appear to be new to economics, although they naturally follow from existing results. In contrast, the work on forward looking systems is entirely new. We defer consideration of systems with arbitrary dimensions to future work, noting here that the 2×2 cases we study capture a range of important contexts, most notably the evaluation of macroeconomic stabilization policy.

In addition to presenting abstract results on frequency specific tradeoffs, we apply our methods to the analysis of monetary policy rules. This work provides a supplement to studies such as Judd and Rudebusch (1998) which focus on regime-specific tradeoffs between overall inflation and output gap volatility. Here we engage in two exercises. First, we examine the frequency by frequency differences in the effects of the monetary policy rules followed by Arthur Burns, Alan Greenspan, and Paul Volcker. As a positive exercise, we are able to show how differences in the inflation and the output gap in the different regimes involve specific tradeoffs frequency by frequency. One example of an insight gained from our approach is that the Volcker regime's major difference from the other two regimes involves the reduction of inflation at the cost of increased low frequency output gap volatility. Second, we demonstrate how one can supplement the conventional inflation/output gap Phillips curve with frequency-specific Phillips curves. The frequency-specific Phillips curves allow one to assess the monetary policy regimes in terms of their efficiency, as measured by their distance from the tradeoff frontier. These exercises, of course, are dependent on an assumed model of the joint proves of inflation and output determination. We in fact consider different models indexed by the role of expectations. The behavior of the rules across different models is also informative. For example, the Volcker low frequency output gap "sacrifice" is inefficient when expectations of future inflation on current inflation are small, but is efficient when these expectations matter.

In addition to providing new positive insights into the structure of tradeoffs between different stabilization objectives, we further believe that frequency-specific tradeoffs have normative interest in evaluating monetary policies. One reaction to the recognition that a Central Bank face frequency-by-frequency constraints might be that these constraints are irrelevant if the objective of a policymaker is to minimize the overall variance of some combination of states and controls of the system, Such loss functions are standard in the literature on evaluating monetary policy rules. We argue that our results are of interest for several reasons. First, there is no principled reason why a Central Bank's loss function should only depend on the overall variances of variables of interest, and in fact time-nonseparable preferences can lead to the assignment of different loss function weights across frequency-specific fluctuations. Examples of this property are found in Otrok (2001, 2002). Second, differences in the approximation value of a given model to fluctuations at different frequencies may lead to a focus on higher versus lower frequency fluctuations using a model to assess policies; this type of reasoning is developed in Onatski and Williams (2003). Third, frequency-specific fluctuations can matter for structural reasons. Meltzer (2003 pp. 65–66) describes how a major reason for the creation of the Federal Reserve in 1914 was the magnitude of seasonal fluctuations associated with the agricultural sector. Diminution of these types of fluctuations thus had distributional consequences in the reduction of risk for farmers. More interesting for contemporaneous issues, as we will see, different monetary policy rules, because of design limits, can reduce overall variance at the expense of enhancing the role of low frequency fluctuations and so provide a different perspective on whether high booms and busts are persistent. Fourth, there are classes of problems for which the frequency restrictions matter, even if loss functions only depend on unconditional variances. Specifically, evaluating the robustness of policy rules in the face of model uncertainty may be facilitated using the constraints we describe; an initial example of such an analysis is Brock and Durlauf (2005).

In our judgment, the most important contribution of this paper is its introduction of the idea that macroeconomic stabilization policies involve tradeoffs that are hidden when a policy is evaluated by calculation of its effects on the variances of the standard macroeconomic aggregates. This kind of result is sometimes also called a "conservation law" or "waterbed" result in the engineering literature. Indeed we will exhibit various conservation laws and waterbed results and illustrate their consequences for a set of two sector macroeconomic models of inflation and the output gap that are commonly used in the macroeconomics literature.

The use of frequency domain methods is not original per se, of course. One classic example is Hansen and Sargent's (1980,1981) use of z-transform methods to translate time domain expectations into the frequency domain and thereby solve for testable restrictions of rational expectations models. Another important contribution is Bowden's (1977) and Whiteman's (1985,1986) work on spectral utility and the frequency domain analysis of the effects of policies; Whiteman's work is close in spirit to ours, although it does not address the issue of frequency-specific tradeoffs. More recently, frequency methods have proven to be important in the development of the growing macroeconomic literature on robustness, cf. Sargent (1999), Kasa (2000), and Hansen and Sargent (2007, Chapter 8). That being said, frequency domain approaches continue to be far less popular than time domain methods for analyzing macroeconomic dynamics. We believe the methods developed here complement these other papers in demonstrating that frequency domain approaches have an important role in

[1] Our description of linear systems owes much to the formulation in Kwakernaak and Sivan (1972, especially Chapter 6) and Skogestad and Postlethwaite (1996), as well as Seron et al. (1997) and various articles, e.g. Chen and Nett (1993, 1995) and Wu and Jonckheere (1992).