Journal of Economic Dynamics & Control 37 (2013) 2979–2997

Contents lists available at ScienceDirect

Journal of Economic Dynamics & Control

journal homepage: www.elsevier.com/locate/jedc

Optimal regime switching and threshold effects

R. Boucekkine[a,b,*], A. Pommeret[c], F. Prieur[d]

[a] Aix-Marseille School of Economics (AMSE), Aix-Marseille University, France
[b] UCLouvain, IRES and CORE, Belgium
[c] University of Lausanne and IREGE – University of Savoie, France
[d] LAMETA, University of Montpellier I and INRA, France

ARTICLE INFO

Article history:
Received 16 May 2012
Received in revised form
22 February 2013
Accepted 22 August 2013
Available online 31 August 2013

Keywords:
Multi-stage optimal control
Threshold effects
Irreversibility
Non-renewable resources
Backstop technology

JEL classification:
Q30
Q53
C61
O33

ABSTRACT

We consider a general control problem with two types of optimal regime switch. The first one concerns technological and/or institutional regimes indexed by a finite number of discrete parameter values, and the second features regimes relying on given threshold values for given state variables. We propose a general optimal control framework allowing to derive the first-order optimality conditions and in particular to characterize the geometry of the shadow prices at optimal switching times (if any). We apply this new optimal control material to address the problem of the optimal management of natural resources under ecological irreversibility, and with the possibility to switch to a backstop technology.

1. Introduction

Whilst Markov-switching models are popular and optimal policy under potential changes in regime has been developed (see Hamilton, 1990, for a seminal presentation, and Zampolli, 2006, for a recent application to monetary policy), these are the problems where the optimal response to an uncertain regime is derived. This paper is about the regime switch as *an optimal response* to explicit or implicit trade-offs. In short, while the well-known Markov-switching models are concerned with optimal policies to deal with given random regime changes, switching is considered itself as a decision variable in our framework, therefore expanding the set of policy options. Typically, regimes refer to institutional and/or technological states of the world. For example, an economy starting with a given technology might find it optimal to switch to a newly available technology or to stick to the old one. Similarly, an economy initially in autarky might decide to switch to full or partial financial liberalization letting international assets flow in and out. More institutional examples can be easily picked: an economy initially out of international agreements (like the Kyoto Protocol) might decide to join them or remain out forever.

* Corresponding author at: Aix-Marseille School of Economics (AMSE), Aix-Marseille University, France.
E-mail addresses: raouf.boucekkine@uclouvain.be (R. Boucekkine), aude.pommeret@unil.ch (A. Pommeret), prieur@supagro.inra.fr (F. Prieur).

0165-1889/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.jedc.2013.08.008

In all cases, the corresponding switching decision is shaped by the inherent nontrivial trade-offs under scrutiny. For example, in the first technological example, the superiority of a newly available technology does not necessarily lead to immediate switching to this technology (or immediate adoption) because of the associated obsolescence and learning costs (see Parente, 1994, or Boucekkine et al., 2004). In the second and more institutional example, switching to full financial liberalization is beneficial because it brings more resources to the economy, but it also induces a clear cost through the external debt burden (see Makris, 2001).

Since the mid 1980s, a substantial optimal control literature has been devoted to handle the class of optimal switching problems described above. This literature is mostly concerned with deterministic setting. We shall consider the same framework in this paper.[1] Tomiyama (1985) and Amit (1986) are, to our knowledge, the earliest contributors to the related optimal control literature. Interestingly enough, these two authors reformulate the optimal switching problem as an optimal timing problem, therefore introducing the time of switch as an explicit decision variable. Immediate switching and sticking to the initial regime (or never switching) correspond to the corner solutions of the problem in this setting. The authors have accordingly developed the maximum principle fitting the general optimal switching problems considered, with a special attention to the continuity/differentiability properties of the shadow prices and Hamiltonians at the switching times (if any). Extensions of the maximum principle developed to any finite number of switching dates (corresponding to any finite number of successive technological and/or institutional regimes) have been already provided (see for example Saglam, 2011). Finally, dynamic games settings encompassing optimal switching problems *à la* Tomiyama have been recently considered (see Boucekkine et al., 2011).

A common feature of all the problems studied in the literature outlined above is that the technological and/or institutional regimes are differentiated through a finite number of discrete parameter values. For example, a newly available technological regime may exhibit a higher productivity parameter but a larger abatement cost parameter compared to the currently used technology. But one can think of many economic problems in which another type of regime switching is at work. Regime switching now is related to the notion of threshold. Thresholds are defined as critical values of a state variable with the very characteristic that crossing a threshold induces a regime change. To the authors' knowledge, the main application of optimal control problems dealing with this kind of regime switching involves the issue of irreversible pollution (see and Prieur et al., in press).[2] There are however many other possible applications of control problems with regime switching induced by the crossing of thresholds. One obvious example pertains to development and growth. For example, in the spirit of Azariadis and Drazen (1990), one may define a threshold on human capital that determines the returns to education. When demography comes into the picture, one may also consider the existence of minimal population size for economic take-off (see Galor, 2005), etc. If these problems differ by nature from optimal regime switching problems *à la* Tomiyama, it has been shown that they can be formulated as optimal timing problems, the date at which the threshold value is reached (if any) being again an explicit control variable.

The aim of this paper is to consider the general problem where both types of regime switching co-exist. It is easy to understand why such problems are highly relevant from the economic point of view. Consider the ecological problem with irreversibility described above and allow the economy to also decide about whether to switch to a newly available technology, cleaner but less productive than the pre-existing one. Clearly enough, the two switching problems, the ecological and the technological optimal switching problems, will "interact". Indeed, the possibility to choose (at a certain optimal date) a cleaner technology might decisively shape the decision to go or not for an ecological switch. To our knowledge, the unique paper considering this type of problems is Boucekkine et al. (2013). The authors show that the interaction of two types of switching problems may generate a wide variety of relationships between pollution and capital (as a proxy for output), mostly inconsistent with the environmental Kuznets curve.

This paper makes two contributions. First, it proposes a general appraisal of optimal switching problems involving both types of regimes: technological and/or institutional regimes indexed by a finite number of discrete parameter values, and ecological-like regimes which rely on given threshold values for given state variables. In this sense, we generalize Boucekkine et al. (2013). The proposed general optimal control framework allows to derive the first-order optimality conditions and in particular to characterize the continuity and differentiability of the shadow prices and Hamiltonians at optimal switching times. This is done using standard optimal control techniques and we do obtain a clear-cut characterization of how the optimal solutions look like in this sophisticated control framework.[3] A second contribution of this paper is to apply this new optimal control material to address the problem of the optimal management of natural resources under ecological irreversibility and the possibility to switch to a backstop technology. For that purpose, we extend the classical exhaustible-resource/stock-pollution model, studied by Tahvonen (1997), by introducing the two types of regime switchings discussed so far. In addition to the ingredient of irreversible pollution to gather the ecological switch into the analysis, the economy has also the possibility to switch to a cleaner backstop technology in the same vein as

[1] Optimal switching problems under uncertainty can be found in the literature, see for example Pommeret and Schubert (2009).

[2] Consider the standard pollution problem where Nature absorbs part of the pollution stock, giving rise to the so-called rate of natural decay of pollution. Irreversibility comes to the story here as follows: when the pollution stock exceeds a certain threshold value, the natural decay rate goes down permanently, that is in an irreversible way. Put differently the system switches to another ecological regime.

[3] An alternative would have been to use more complex techniques belonging to hybrid control theory as in Shaikh and Caines (2007) commonly used in engineering. Given the specificity of the general hybrid problems involved in economics (see next section), our approach seems more natural and more accessible to the economists community.