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Expectational stability of sunspot equilibria in non-convex economies

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ABSTRACT

We examine the stability under learning (E-stability) of sunspot equilibria in non-convex real business cycle models. The production technology is Cobb–Douglas with externalities generated by factor inputs. We establish that, with a general utility function, the well-known Benhabib–Farmer condition (Benhabib and Farmer, 1994) – that the labor-demand curve is upward-sloping and steeper than the Frisch labor-supply curve – is necessary for joint indeterminacy and E-stability. Then, with a separable utility function and allowing for negative externalities from capital inputs, we discover large regions in parameter space corresponding to stable indeterminacy, that is, learnable sunspot equilibria. These existence results overturn the conventional wisdom that sunspot equilibria in RBC-type models are inherently unstable, and provide concise closure to the *stability puzzle* of Evans and McGough (2005b).

1. Introduction

Work by Shell (1977), Cass and Shell (1983), Azariadis (1981), and others, demonstrated the potential for macroeconomic models, including those couched in general equilibrium theory, to be indeterminate, and thus to exhibit equilibrium outcomes that depend on extrinsic shocks, i.e. shocks that affect the economy *only* because the economy's agents expect them to: these shocks are called sunspots and the associated equilibrium outcomes are called sunspot equilibria. Naturally interpreted as self-fulfilling prophecies, these sunspot equilibria gave rise to a new explanation of the business cycle: fluctuations in economic activity may be the result of variation in agents' beliefs.

Separately, the literature on learning in macroeconomics developed in part as a justification for the strong informational assumptions required to support the rational expectations hypothesis. In a learning environment, agents are assumed to form expectations using the same types of econometric forecasting models as economists. If the resulting endogenous variables converge to a rational expectations equilibrium (REE), then that equilibrium is said to be stable

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under learning, and the case for relying on rational expectations as a modeling assumption, and for focusing attention on the REE, is correspondingly strengthened.

Stability under learning is not generic. Even for a given functional structure, some parameterizations of a model may yield learnable equilibria while for others the equilibria may be unstable. In this way, learnability may be viewed as a selection criterion: the relevance of a particular model's implications may be strengthened if the associated equilibria are stable under learning. Via this notion, Woodford (1990) lent additional credence to the sunspot explanation of the business cycle by showing in a simple overlapping generations model that the associated sunspot equilibria are stable.

With the pioneering work of Benhabib and Farmer (1994) and Farmer and Guo (1994), sunspot driven business cycles were joined to applied dynamic stochastic general equilibrium (DSGE) modeling. These authors developed calibrated non-convex real business cycle (RBC) models that well-matched the data using only sunspot processes as exogenous stochastic drivers. Their work spawned a large and still growing literature dedicated to exploring and relaxing conditions under which RBC-type and other DSGE models exhibit sunspot equilibria.

As mentioned above, stability under learning is model dependent, and while Woodford (1990) found that sunspot equilibria *may* be stable under learning, he did not show that they have to be stable; and, in fact, Evans and Honkapohja (2001) found that the equilibria analyzed by Farmer and Guo (1994), at least for the particular calibration used, are not stable under learning. This instability result was explored in detail by Evans and McGough (2005b): they studied multiple parameterizations of a variety of RBC-type models and found no stable sunspot equilibria: they dubbed the contrast of their instability results with Woodford's stability results " the stability puzzle."[1]

Unraveling the stability puzzle has proven to be complicated. Evans and McGough (2005b, 2010) emphasize the representation dependence of stability under learning. A given sunspot equilibrium may be characterized by a number of natural recursions, or "representations," depending on what type of sunspot process is taken as observable by agents; and, the chosen recursion dictates the functional form of the forecasting model used by learning agents when forming expectations. Further, it can be shown that whether a given equilibrium is stable under learning may depend on the agents' forecasting model, and thus on the representation used by the researcher when conducting stability analysis. Evans and McGough (2010) showed that Woodford (1990) and Evans and Honkapohja (2001) used fundamentally different representation types, which could, in principle, explain their differing stability results; however, using the same representation type as Woodford (1990), Evans and McGough (2005b) still failed to find stable sunspot equilibria.[2] Duffy and Xiao (2007) also studied the stability puzzle. They emphasized a particular restriction on the reduced form parameters necessary for stable sunspots – that the coefficient modifying expectations of future aggregate consumption be negative – and then showed that the models examined by Evans and McGough (2005b) never met this condition.

While the work described above represents important progress towards understanding the stability puzzle, the fundamental question remains: are there non-convex RBC models which exhibit stable sunspot equilibria? The central contribution of this paper is to demonstrate that the answer to this question is an unqualified " yes." We take, as our point of departure, a discrete-time version of the model studied by Meng and Yip (2008), which may be interpreted as the Benhabib–Farmer–Guo model extended to incorporate a general utility function and possibly negative capital externalities. We begin studying the restrictions on the model's deep structure necessary for sunspot equilibria to be stable under learning. We find the generic condition that if stable sunspot equilibria exist then the labor-demand curve crosses the Frisch labor-supply curve from below.[3]

By requiring that utility is separable we obtain additional conditions for learnable sunspots: the marginal effect of capital on the externality must be negative and the steady-state relative risk aversion in consumption must be less than unity. Using these necessary restrictions to provide guidance, we determine that in large regions of the model's parameter space, stable sunspot equilibria exist.

The organization of this paper is as follows. In Section 2 we present the Benhabib–Farmer–Guo model modified to incorporate a general utility function and possibly negative capital externalities. Here, we also provide the needed background on indeterminacy and adaptive learning, placing particular emphasis on the importance of forecast-model specification (equilibrium representation). Section 3 reports the analytic and numeric results, including subsections detailing the conditions necessary for learnable sunspots and numerical evidence of their existence, and with Fig. 1 succinctly summarizing the results for the case with separable utility (which are also the main results of this paper). In Section 4, we then provide a detailed discussion of several important issues raised by our results; these issues include the plausibility of stable sunspot equilibria, alternative equilibrium notions, and real-time learning. Section 5 concludes.

[1] The stability puzzle involves the additional observation that the a-theoretic reduced-form system of expectational difference equations (see Eqs. (14) and (15)) associated to non-convex RBC models of the form studied by Evans and McGough (2005b) does allow for stable sunspot equilibria: it is only when the reduced form parameters are derived from an underlying DSGE model that instability appears to necessarily obtain. See Section 2.3.4 for details.

[2] Using an alternative timing convention, Evans and McGough (2005b) did find small regions of the model's parameter space corresponding to stable sunspots: see Section 4.1 for details.

[3] In their model with positive capital externalities, Benhabib and Farmer (1994) established this condition as necessary for the *existence* of sunspot equilibria. Meng and Yip (2008) found that provided the capital externality is negative, sunspot equilibria may exist even if labor demand can be downward sloping. Here we find that the Behabib–Farmer condition re-emerges even with negative capital externalities if we insist upon learnability: see Section 3.1 for details.