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Zipf's law and maximum sustainable growth

Y. Malevergne [a,b,c,*], A. Saichev [c,d], D. Sornette [c,e]

[a] Université de Lyon, Université de Saint-Etienne, Coactis E.A. 4161, France
[b] EMLYON Business School, Cefra, France
[c] ETH Zurich, D-MTEC, Switzerland
[d] Nizhny Novgorod State University, Department of Mathematics, Russia
[e] Swiss Finance Institute, Switzerland

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ABSTRACT

Zipf's law states that the number of firms with size greater than S is inversely proportional to S. Most explanations start with Gibrat's rule of proportional growth but require additional constraints. We show that Gibrat's rule, at all firm levels, yields Zipf's law under a balance condition between the effective growth rate of incumbent firms (which includes their possible demise) and the growth rate of investments in entrant firms. Under the additional assumption that firms do not consume more resources than available, we show that Zipf's law is the signature that firms grow at the maximum reachable long-term rate.

1. Introduction

The relevance of power law distributions of firm sizes to help understand firm and economic growth has been recognized early, for instance by Schumpeter (1934), who proposed that there might be important links between firm size distributions and firm growth. The endogenous and exogenous processes and the factors that combine to shape the distribution of firm sizes can be expected to be at least partially revealed by the characteristics of the distribution of firm sizes. The distribution of firm sizes has also attracted a great deal of attention in the recent policy debate (for instance Eurostat, 1998), because it may influence job creation and destruction (Davis et al., 1996), the response of the economy to monetary shocks (Gertler and Gilchrist, 1994) and might even be an important determinant of productivity growth at the macroeconomic level due to the role of market structure (Peretto, 1999; Pagano and Schivardi, 2003; Acs et al., 1999).

This paper presents a reduced form model that provides a generic explanation for the ubiquitous stylized observation of power law distributions of firm sizes, and in particular of Zipf's law—i.e., the fact that the fraction of firms of an economy whose sizes S are larger than s is inversely proportional to s: $\Pr(S>s) \sim s^{-m}$, with m equal (or close) to 1. We consider an economy made of a large number of firms that are created according to a random birth flow, disappear when failing to remain above a viable size, go bankrupt when an operational fault strikes, and grow or shrink stochastically at each time step proportionally to their current sizes (Gibrat's law).

* Corresponding author at: Université de Lyon, Université de Saint-Etienne, Coactis E.A. 4161, France.
E-mail addresses: ymalevergne@ethz.ch (Y. Malevergne), saichev@hotmail.com (A. Saichev), dsornette@ethz.ch (D. Sornette).

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Our contribution to the ongoing debate on the shape of the distribution of firms' sizes is to present a theory that encompasses previous approaches and to derive Zipf's law as the result of the combination of simple but realistic stochastic processes of firms' birth and death together with Gibrat's law (Gibrat, 1931) law. The main result of our approach is that Zipf's law is obtained if and only if the firm sizes grow at the maximum expected rate under a balance condition between the growth rate of available external resources and the growth rate of the economy due to the reallocation of the resources freed by the failing firms. Another interesting aspect of our framework is the analysis of deviations from the pure Zipf's law (case $m=1$) under a variety of circumstances resulting from transient imbalances between the average growth rate of incumbent firms and the growth rate of external resources. These deviations from the pure Zipf's law have been documented for a variety of firm's size proxies (e.g. sales, incomes, number of employees, or total assets), and reported values for m ranges from 0.8 to 1.2 (Ijiri and Simon, 1977; Sutton, 1997; Axtell, 2001, among many others). Our approach provides a framework for identifying their possible (multiple) origins.

In the literature on the growth dynamics of business firms, a well established tradition describes the change of the firm's size, over a given period of time, as the cumulative effect of a number of different shocks originated by the diverse accidents that affected the firm in that period (Kalecki, 1945; Ijiri and Simon, 1977; Steindl, 1965; Sutton, 1998; Geroski, 2000, among others). This, together with Gibrat's law of proportional growth, forms the starting point for various attempts to explain Zipf's law. However, these attempts generally start with the implicit or explicit assumption that the set of firms under consideration were born at the same origin of time and live forever (Gibrat, 1931; Gabaix, 1999; Rossi-Hansberg and Wright, 2007a,b). As a consequence, the distribution of firm sizes reaches a steady-state if and only if the distribution of the size of a single firm reaches a steady state. This latter assumption is counterfactual or, even worse, non-falsifiable.

An alternative approach to model a stationary distribution of firm sizes is to account for the fact that firms do not all appear at the same time but are born according to a more or less regular flow of newly created firms, as suggested by common sense.[1] Simon (1955) was the first to address this question (see also Ijiri and Simon, 1977). He proposed to modify Gibrat's model by accounting for the entry of new firms over time as the overall industry grows. He then obtained a steady-state distribution of firm sizes with a regularly varying upper tail whose exponent m goes to one from above, in the limit of a vanishingly small probability that a new firm is created. This situation is not quite relevant to explain empirical data, insofar as the convergence toward the steady-state is then infinitely slow, as noted by Krugman (1996). More recently, Gabaix (1999) allowed for birth of new entities, with the probability to create a new entity of a given size being proportional to the current fraction of entities of that size and otherwise independent of time. In fact, this assumption does not reflect the real dynamics of firms' creation. For instance, Bartelsman et al. (2005) document that entrant firms have a relatively small size compared with the more mature efficient size they develop as they grow. It seems unrealistic to expect a non-zero probability for the birth of a firm of very large size, say, of size comparable to the largest capitalization currently in the market.[2] In this respect, Luttmer's (2007) model is more realistic than Gabaix's, insofar as it considers that entrant firms adopt a scaled-down version of the technology of incumbent firms and therefore endogenously set the size of entrant firms as a fraction of the size of operating firms. In this paper, we partly follow this view and consider that the size of entrant firms is smaller than the size of incumbent firms. But we depart from Luttmer's because the size of new entrants is not endogenously fixed in our model. We set this parameter exogenously for versatility reasons.

Another key ingredient characterizes our model. The fact that firms can go bankrupt and disappear from the economy is a crucial observation that is often neglected in models. Many firms are known to undergo transient periods of decay which, when persistent, may ultimately lead to their exit from business (Bonaccorsi Di Patti and Dell'Ariccia, 2004; Knaup, 2005; Brixy and Grotz, 2007; Bartelsman et al., 2005). Simon (1960) as well as Steindl (1965) have considered this stylized fact within a generalization of Simon (1955), where the decline of a firm and ultimately its exit occurs when its size reaches zero. In Simon's (1960) model, the rate of firms' exit exactly compensates the flow of firms' births so that the economy is stationary and the steady-state distribution of firm sizes exhibit the same upper tail behavior as in Simon (1955). In contrast, Steindl (1965) includes births and deaths but within an industry with a growing number of firms. A steady-state distribution is obtained whose tail follows a power law with an exponent that depends on the net entry rate of new firms and on the average growth rate of incumbent firms. Zipf's law is only recovered in the limit where the net entry rate of new firms goes to zero. Both models rely on the existence of a minimum size below which a firm runs out of business. This hypothesis corresponds to the existence of a minimum efficient size below which a firm cannot operate, as is well established in economic theory. However, there may be in general more than one minimum size as the exit level of a firm has no reason to be equal to the size of a firm at birth. In the aforementioned models, these two sizes are assumed to be equal, while there is *a priori* no reason for such an assumption and empirical evidence *a contrario*. In our model, we allow for two different thresholds, the first one for the typical size of entrant firms and the second one for the exit level. This second level is assumed to be lower than the first one, even if recent evidence seems to suggest that firms might enter with a size less than their minimum efficient size (Agarwal and Audretsch, 2001) and then rapidly grow beyond this threshold in order to survive.

[1] See Dunne et al. (1988), Reynolds et al. (1994) or Bonaccorsi Di Patti and Dell'Ariccia (2004), among many others, for "demographic" studies on the populations of firms.

[2] We do not consider spin-off's or M&A (mergers and acquisitions).