Computers and Structures 173 (2016) 19–30

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Non-linear global stability analysis of thin-walled laminated beam-type structures

Igor Pesic *, Domagoj Lanc, Goran Turkalj

Department of Engineering Mechanics, Faculty of Engineering, University of Rijeka, Vukovarska 58, HR-51000 Rijeka, Croatia

ARTICLE INFO

Article history:
Received 25 May 2015
Accepted 15 May 2016
Available online 8 June 2016

Keywords:
Stability
Buckling
Composite
Laminate
Critical load
Creep

ABSTRACT

The paper presents an algorithm for buckling analysis of thin-walled laminated composite beam-type structures. One-dimensional finite element is employed under the assumptions of large displacements, large rotation effects but small strains. The equilibrium equations of a prismatic and straight spatial beam element are formulated using the virtual work principle. Stability analysis is performed in load deflection manner using corotational formulation. The cross-section mid-line contour is assumed to remain not deformed in its own plane, whereas the shear strains of middle surface are neglected. Laminates are modelled on the basis of classical lamination theory. Results have been validated on test examples.

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1. Introduction

Composite materials are very suitable for structural applications where both high strength to weight and stiffness to weight ratios are required. Fibre reinforced laminates in the form of thin-walled beams have been increasingly used during the past few decades in engineering practice, particularly in aerospace engineering, shipbuilding, civil engineering and in the automobile industry. Such weight-optimized structural components are commonly very susceptive to buckling failure because of their slenderness, so stability problems should be considered in their design [1].

Bauld and Tzeng [2] have extended Vlasov's theory of bending and twisting of thin-walled composite beams with an open cross-section made from symmetric fibre-reinforced laminates. Composite beams with arbitrary geometric and material sectional properties have been studied in [3,4]. Bhaskar and Librescu [5] have presented a geometrically non-linear theory of composite thin-walled beams accounting for finite flexural displacements and an arbitrarily large twist angle. Lee and Kim have presented an analytical model which accounts for flexural–torsional buckling of I-section composite beams [6] and an analytical model which accounts for lateral buckling of thin-walled laminated channel-section beams [7].

Cardoso et al. [8] have presented a finite element model for structural analysis of composite laminated thin-walled beam structures with geometrically non-linear behaviour, including post-critical behaviour and warping deformation. Vo et al. [9,10] analysed flexural–torsional coupled buckling of thin-walled composite beams with arbitrary lay-ups. Silvestre, Camotim and coworkers [11–13] have presented the formulation of a second-order Generalised Beam Theory (GBT) developed to analyse the buckling behaviour of composite thin-walled members and taking into account both local and global deformation modes.

Barbero and Luciano [14] developed a micromechanical model to characterize linear viscoelastic solids with periodic microstructure. Oliveira and Creus [15] have developed a study of viscoelastic thin-walled straight beams by means of a non-linear approach using finite shell elements. Piovan and Cortinez [16] have constructed a study on the linear viscoelastic behaviour of thin-walled curved and straight beams with composite materials of polymeric matrix.

The stability analysis can be performed using two different approaches. In the first case, the stability analysis is performed in an eigenvalue manner, which allows us to determine the instability load of the structure in a direct manner without calculating the exact magnitude of deformations. The lowest eigenvalue obtained is recognized as the critical or buckling load and the corresponding eigenvector the shape of buckling. In the second case, the stability problems are investigated using the load–deflection manner [17,18] by which the structural behaviour throughout the entire range of loading of interest is evaluated, including the pre-buckling and maybe post-buckling phase. That approach, known as the non-linear stability analysis, when compared with the linearized one provides information more reliably for imperfect or

* Corresponding author.

http://dx.doi.org/10.1016/j.compstruc.2016.05.015
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real structures and loading conditions with or without material non-linearity, for which the eigenvalue approach generally gives overestimated results. The non-linear response of a load-carrying structure should be solved using numerical methods, e.g., the finite element method and some of incremental descriptions like the total and updated Lagrangian ones, respectively, or the corotational description [19]. Each description utilizes a different structural configuration for system quantities referring to and results as a set of non-linear equilibrium equations of the structure. This set can further be linearized and should be solved using some incremental–iterative scheme.

In the Ref. [20], the same authors presented a non-linear beam model based on the updated Lagrangian (UL) incremental description and non-linear cross-section displacement field which accounts for the second order displacement terms due to large rotations. Turkalj et al. [21] expanded that model for beam type structures with semi-rigid connections. Unlike previous papers [20,21], this paper uses the corotational formulation as a well-known approach to the development of efficient beam elements for non-linear analysis of structures [22] and it deals with the viscoelastic behaviour of composite laminated fibre-reinforced plastic beams.

The corotational description that has been used in this paper is linear on element-level and all geometrically non-linear effects are introduced through the transformation from the local coordinate system to the global one. The local corotational system follows the element chord during the deformation and allows the usage of simplified strain–displacement relations on the local element-level [23–25].

A finite element model for stability analysis of 3D framed structures with thin-walled laminated composite cross-section is presented in this paper. The beam cross-section geometry is discretized by quadratic monitoring areas and the structural discretization is performed by the space beam finite element. The model takes into account effects of large displacements on the response of space frames subjected to conservative and static external loads. The shear strain in the middle surface is assumed to be zero and the cross-section is not distorted in its own plane.

Classical lamination theory of thin fibre-reinforced laminates has been employed. The model is applicable to any arbitrary laminate cross-section shape. Verification examples utilizing a numerical algorithm developed on the basis of abovementioned procedure have been presented to demonstrate the accuracy of this model [26]. It seems that examples dealing with the space frames where large rotations come to the fore have been poorly treated in the open literature. The present model is found to be appropriate and efficient in analysing complex structural behaviour under a large displacement and rotation regime.

2. Theoretical background

2.1. Kinematics

Initially straight composite thin-walled beam with an arbitrary but undeformable cross-section is considered. The points of the structural member refer to the local Cartesian coordinate system in which the beam axis that connects all cross-sectional centres of gravity coincides with the z axis, while the x and y are the axes of the cross-section, but not necessarily the principal ones. Additionally, a circumferential coordinate s and a normal coordinate n are introduced into the middle contour of the cross-section. Cross-sectional rigid-body displacements are:

$$w_0 = w_0(z), \quad u_0 = u_0(z), \quad v_0 = v_0(z),$$
$$\varphi_z = \varphi_z(z), \quad \varphi_x = -\frac{\mathrm{d}v_0}{\mathrm{d}z}, \quad \varphi_y = \frac{\mathrm{d}u_0}{\mathrm{d}z}, \quad \theta = -\frac{\mathrm{d}\varphi_z}{\mathrm{d}z}. \tag{1}$$

In the above equations w_0, u_0 and v_0 are the rigid-body translations in the z, x and y directions, respectively; while φ_z, φ_x and φ_y are the rigid-body rotations around z, x and y axes, respectively. Displacement θ is a cross-sectional warping parameter.

Assuming that displacements and rotations are small in local corotational coordinate system, the displacement components of an arbitrary point of the cross-section can be expressed as:

$$\begin{aligned} w &= w_0 - y\frac{\mathrm{d}v_0}{\mathrm{d}z} - x\frac{\mathrm{d}u_0}{\mathrm{d}z} - \omega\frac{\mathrm{d}\varphi_0}{\mathrm{d}z}, \\ u &= u_0 - y\,\varphi_z, \\ v &= v_0 + x\,\varphi_z \end{aligned} \tag{2}$$

where x and y define the position of the cross-section, while ω is a value of the cross-sectional warping function. Since it is assumed that $\varepsilon_{zn} = 0$, the strain tensor ε contains only two components, i.e.,

$$\boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_z \\ \varepsilon_{zs} \end{Bmatrix} \tag{3}$$

where from Eqs. (1) and (2) follows that:

$$\varepsilon_z = \frac{\mathrm{d}w_0}{\mathrm{d}z} - y\frac{\mathrm{d}\varphi_x}{\mathrm{d}z} - x\frac{\mathrm{d}\varphi_y}{\mathrm{d}z} - \omega\frac{\mathrm{d}^2\varphi_z}{\mathrm{d}z^2} + \frac{1}{2}(x^2+y^2)\left(\frac{\mathrm{d}\varphi_z}{\mathrm{d}z}\right)^2, \tag{4}$$

$$\varepsilon_{zs} = 2n\frac{\mathrm{d}\varphi_z}{\mathrm{d}z}, \tag{5}$$

First-order approximation of the strain components is employed in the above equations because a corotational approach assumes small displacements in the local coordinate system. The only quadratic term in Eq. (4) is necessary to model Wagner's effect [19,27].

The first variations of the strain tensor can be obtained from Eqs. (4) and (5) as

$$\delta\boldsymbol{\varepsilon} = \mathbf{x}_\varepsilon \delta\tilde{\boldsymbol{\varepsilon}} \tag{6}$$

where

$$\delta\tilde{\boldsymbol{\varepsilon}}^{\mathrm{T}} = \left\{ \frac{\mathrm{d}\delta w_0}{\mathrm{d}z} \quad \frac{\mathrm{d}\delta\varphi_x}{\mathrm{d}z} \quad \frac{\mathrm{d}\delta\varphi_y}{\mathrm{d}z} \quad \frac{\mathrm{d}^2\delta\varphi_z}{\mathrm{d}z^2} \quad \frac{\mathrm{d}\delta\varphi_z}{\mathrm{d}z}\cdot\frac{\mathrm{d}\varphi_z}{\mathrm{d}z} \quad \frac{\mathrm{d}\delta\varphi_z}{\mathrm{d}z} \right\} \tag{7}$$

$$\mathbf{x}_\varepsilon = \begin{bmatrix} 1 & -y & -x & -\omega & (x^2+y^2) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2n \end{bmatrix}. \tag{8}$$

2.2. Internal forces

The stress tensor for the spatial beam element has two components:

$$\boldsymbol{\sigma} = \begin{Bmatrix} \sigma_z \\ \tau_{zs} \end{Bmatrix} \tag{9}$$

Integrating over the laminate thickness n and the contour direction s, and transforming into the beam coordinate system, the cross-sectional internal force components follow as:

$$\begin{aligned} &F_z = \int_A \sigma_z\,\mathrm{d}A, \quad M_x = \int_A \sigma_z y\,\mathrm{d}A, \quad M_y = -\int_A \sigma_z x\,\mathrm{d}A, \\ &M_z = \int_A \tau_{zs} n\,\mathrm{d}A, \quad M_\omega = \int_A \sigma_z \omega\,\mathrm{d}A, \quad T_\sigma = \int_A \sigma_z (x^2+y^2)\,\mathrm{d}A \end{aligned} \tag{10}$$

where F_z represents the axial force, F_x and F_y are shear forces, M_z is the St. Venant torsion moment, M_x and M_y are bending moments with respect to the x and y axes, respectively, M_ω is the bimoment and T_σ is Wagner coefficient [9]. Shear forces are treated as reactive ones so that they can be determined as $F_x = -\mathrm{d}M_y/\mathrm{d}z$ and $F_y = \mathrm{d}M_x/\mathrm{d}z$. Thus, the vector of active internal forces

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