Journal of Economic Dynamics & Control 37 (2013) 2168–2179

Contents lists available at ScienceDirect

Journal of Economic Dynamics & Control

journal homepage: www.elsevier.com/locate/jedc

A family production overlapping generations economy

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ARTICLE INFO

Article history:
Received 19 March 2010
Accepted 22 March 2013
Available online 24 May 2013

JEL classification:
C62
D84
D91

Keywords:
Overlapping generations
Family production
Expectation function

ABSTRACT

This paper provides a theoretical analysis of an overlapping generations economy in which production decisions and input–output allocations are all carried out at the family level. I consider a single class of output allocation schemes and various degrees of knowledge about the production technology. Under complete knowledge, I show that a family organizational structure in which everyone receives his marginal contribution to output, invests less in physical capital than under a perfectly competitive equilibrium environment. Under incomplete knowledge, I analyze and compare how beliefs about the input–output relationship affect the physical capital accumulation dynamics and the long-run standards of living.

1. Introduction

General equilibrium models augmented with finitely lived overlapping generations (OLG) were introduced by Allais (1947), Samuelson (1958), Diamond (1965) and have become very popular[1] frameworks in macroeconomics. Most OLG models rely on the perfectly competitive equilibrium hypothesis[2] and on the assumption that individuals have perfect knowledge about their environment of interest.[3] Samuelson's (1958) analysis of a pure exchange OLG economy revealed that suboptimal perfectly competitive equilibria may arise even in the absence of externalities. In a production OLG model, Diamond (1965) showed that its perfectly competitive equilibrium may lead to either over-accumulation or under-accumulation of physical capital with respect to Phelps' (1961) golden rule steady-state. These suboptimal perfectly competitive equilibria were believed to be caused by missing markets across generations[4] until Shell (1971) identified the theoretical peculiarity of OLG models known as the 'double infinity of traders and dated commodities'[5] as the root cause for the failure of the first fundamental welfare theorem. Since perfectly competitive equilibrium markets may not provide the most efficient resource allocation in this framework, does this result also extend to non-market-based organizational structure?

The current paper departs from the benchmark perfectly competitive equilibrium case by considering an OLG economy in which production decisions and input–output allocations are all carried out at the family level with consumption/

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[1] Due to their relative computational simplicity and explicit representation of the demographic structure.

[2] Many OLG models such as the one by Gloom and Ravikumar (1992) do not explicitly consider competitive markets.

[3] OLG models with fully specified learning schemes have been analyzed by Benassy and Blad (1989), Bullard (1994), Schönhofer (1999), Lettau and Van Zandt (2003), Tuinstra (2003).

[4] Individuals cannot all meet on a single market due to their finite lifetime.

[5] In a discrete time infinitely lived OLG model with a single traded good at every period, there are both a countable infinity of dated commodities and a countable infinity of trading agents.

0165-1889/$ - see front matter
http://dx.doi.org/10.1016/j.jedc.2013.05.009

investment decisions being taken at the individual level. I define a family as a multi-generational household including an old agent or 'parent' along with a young agent that stands for his corresponding 'offspring' and I use the term 'family economy' to refer to the production and allocation structure taking place between these two family members. In the pre-industrialized era, the predominance of family production has been emphasized by many economic historians such as Atack et al. (2000), Margo (2000), Ruggles (2001), Carter et al. (2003). Was this organizational structure one of the main reasons behind the slow rate of physical capital accumulation during that time period in history? Among economists, the theme of family production has not received the same level of attention as household production[6] with the exception of Lord and Rangazas (2006) who calibrated the U.S. and England an OLG model including both a family and a standard formal sector[7] to reproduce trends in fertility, schooling, and rate of growth. The negative effects of a family production structure on growth have been emphasized by Caselli and Gennaioli (2013) who pointed out its inefficiencies due to the absence of merit in the ownership and management and by Lord and Rangazas (2006) who highlighted the lesser need to save for retirement of each of its members. Since individuals' choices in a family production environment depend on how the family output is produced and allocated across members, I consider a given class of income allocation schemes based on current output and a set of beliefs about the production process. I show that an economy with a family organizational structure in which everyone has complete knowledge about the production technology and is paid his marginal contribution to output, invests less in physical capital than under a perfectly competitive equilibrium environment. This result is due to the 'monopoly power' given by a family organizational structure to one generation over the supply of this production factor. I also compare the efficiency of the resulting steady-state family production solution with the corresponding perfectly competitive equilibrium outcome. In the situation in which individuals have incomplete knowledge about the production technology, I assume that one-period ahead output forecasts are derived from an expectation function satisfying certain properties as in Grandmont (1985, 1998), Grandmont and Laroque (1986, 1990, 1991), and Böhm and Wenzelburger (1999). For simplicity, I consider that the production expectation function is based on the expected physical capital and observable data as in Dawid (2005). Such representation may for instance encompass the extrapolative adaptive forecasting case derived from a first-order linear approximation of the technology used by Day (2000) and Dawid and Day (2007). If individuals rely on a production expectation function which over-estimates (under-estimates) the steady-state marginal product of the physical capital, then the resulting steady-state is larger (smaller) than under complete knowledge. If individuals correctly predict the steady-state marginal product of the physical capital but under-estimate the rate at which it diminishes, then either higher speeds of convergence than under complete knowledge or overshooting phenomena may occur. For some allocation schemes of the family output and/or beliefs about the production technology, I show using simple numerical examples that either over-accumulation or under-accumulation of physical capital with respect to its golden rule value can be prevented.

The paper is organized as follows. In Section 2, I present a family production economy in which individuals have complete knowledge about the production process. In Section 3, I compare the family allocations with the benchmark perfectly competitive equilibrium case. In Section 4, I analyze how incomplete knowledge about the production technology affects the physical capital accumulation in the family production structure. The last section concludes the paper.

2. Family production under complete knowledge

Let us consider an infinitely lived overlapping generations economy with production and physical capital accumulation. At every time $t \in \mathbb{Z}_+$,a new generation of identical individuals living for two periods denoted by 1 and 2 is born. In period 1, each individual allocates 1 unit of time to the production of a normal and perfectly divisible consumption commodity. Without loss of generality, I consider that population is constant and that each generation size is normalized to 1. Therefore, the economy is populated at every time t by either one worker or one family composed of two members namely a 'parent' born at time $t-1$ and his corresponding 'offspring' born at time t.[8] If the produced commodity is not consumed,then it can be converted into next period's physical capital or perishes otherwise. I assume for simplicity a one-to-one relationship between the amount of consumption commodity invested into physical capital and the amount of this production factor obtained after one period. Without loss of generality and to simplify the model,I consider that the physical capital fully depreciates[9] within each period. Individual lifetime preferences are described by a utility function denoted by U which depends on both first and second-period consumptions: c_1,c_2,respectively.

Assumption 1. The lifetime utility function $U(c_1, c_2) : \mathbb{R}_+^2 \to \mathbb{R}$ is twice continuously differentiable,strictly quasiconcave, increasing in both variables and guarantees that starvation is avoided: $\lim_{c_1 \to 0} U_1(c_1, c_2) = +\infty$ for all $c_2 > 0$, $\lim_{c_2 \to 0} U_2(c_1, c_2) = +\infty$ for all $c_1 > 0$.

[6] See Becker (1965), Polak and Watcher (1975), Gronau (1977). Examples of life cycle models with household production can be found in Benhabib et al. (1991), Greenwood and Hercowitz (1991), Nosal et al. (1992), Fisher (1997), and Rios-Rull (1993).

[7] Each sector produces the same commodity using a different technology.

[8] Under the alternative assumption that the size of the generation born at time t is $N_t \in \mathbb{R}_+$ with $N_t = (1+n)N_{t-1}$ where $n \in (-1, +\infty)$ denotes the population growth rate, then N_{t-1} would stand for the number of families at time t. Each of them would be composed of 2+n members composed of 1 parents and his 1+n corresponding offspring(s).

[9] Since one period usually stands for decades in a two-period OLG model, full physical capital depreciation is often considered (see, e.g., De la Croix and Michel, 2002 in Footnote 7 on page 4). Therefore, no further assumption is required to describe the fate of the remaining unit(s) of physical capital.