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Correlated risks vs contagion in stochastic transition models

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ABSTRACT

This paper studies the problem of disentangling risk correlation and contagion in a set of individual binary processes. The two admissible values correspond to bad and good risk states of an individual. The risk correlation is captured by introducing a dynamic frailty, whereas the contagion passes through the effect of the lagged number of individuals in the bad risk state. We study carefully the dynamic properties of the joint process. Then, we focus on the limiting case of large populations (portfolios). The difficulty to identify risk correlation and contagion in finite samples is illustrated by means of Monte-Carlo simulations.

1. Introduction

The Law of Large Numbers is often invoked to justify the possibility to eliminate, or at least diminish, the risk of a portfolio by diversifying the type of investments. Typically, it was often considered that a large portfolio of individual life insurance contracts, or of mortgages, is almost riskfree, and might be priced by considering its expected value, called pure premium in Insurance. However, the possibility of diversification supposes that the assumptions of the Law of Large Numbers are satisfied, in particular, the fact that the individual risks are not too dependent. The 2007–2009 financial crisis showed that the risk dependencies cannot be neglected and constitute a major component of the risk of the system when computing the reserves of a financial institution. The role of some dependencies was already highlighted in Basel 2 and Solvency 2 regulations (Basel Committee on Banking Supervision, 2001, 2003) beginning to be implemented before the crisis. These were the dependencies due to observable and unobservable common risk factors, called systematic factors and frailties, respectively. For instance, individual mortgages are often contracts with adjustable rates and monthly payments indexed to some prime rate. An increase of this prime rate implies an increase of the level of monthly payments for a lot of mortgages and, thus, correlated defaults. Similarly, the risk of life insurance contracts depends on the general uncertain increase of human lifetime, called longevity risk; this unobservable longevity risk is a major source of possible losses for a life insurance company.

After 2008, the financial regulation has been extended in two directions. First, it highlights contagion as a major source of risk dependencies. Second, it also focuses on the risk of the banking (financial) system as a whole, and not only on the

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separate analysis of the risks of banks and insurance companies. The contagion effect is due to the interconnection between banks and insurance companies by means of their debt and reinsurance structures. Typically, the failure of a company will imply losses for its lenders, and maybe the failure of some of them, then possibly failure of some lenders of the lenders, and so on. We get a propagation of the risk along a chain of failures.

It is clearly important to disentangle the two types of dependencies, namely the effects of shocks exogenous to the system and the contagion phenomenon, which is system endogenous, in order to measure their relative magnitude and to clarify their distinct roles. This distinction is crucial when forecasting the risk of losses incurred by the financial institutions in the system, in particular for the computation of the reserves needed to protect the whole system, i.e., the reserves for systemic risk, and for the definition of the contribution of each financial institution to these reserves for systemic risk. There exist alternative approaches to identify these two components of risk dependencies, which are the structural and reduced form approaches, respectively.

(i) The structural approach tries to get the precise knowledge of all possible connections between the institutions of interest. Typically, if we consider the European banking system, the idea is to know regularly the detailed balance sheets of all the banks, the proportion of debt of each bank with respect to each other ones, for each time to maturity and seniority level, etc. Then, we can construct the contagion network, and understand what might be the propagation in the system of an adverse shock. This is the principle of the so-called stress tests.[1] However, this approach requires databases on the detailed balance sheets, which exist for some countries, but are submitted to strict confidentiality restrictions. Moreover, the complexity of such a network increases very quickly with the number of institutions in the system. This explains why the first studies of this type focus on rather small networks of institutions (see e.g. Egloff et al., 2007), or countries with a sufficiently concentrated banking system (see e.g. Upper and Worms, 2004, for Germany, or Gourieroux et al., 2012, for France). These studies are concerned with solvency risk. In the spirit of Basel 3, the analysis is extended to disentangle the short and long term components of the balance sheet. The short term exposures are used to analyze the potential need for cash and the contagion associated with funding liquidity risk. It seems difficult to apply these approaches directly to the US, for instance, due to the thousands of US financial institutions. When the connections, i.e. the contagion channels, are known, we can consider the effect of (common) exogenous shocks. The exogenous shocks can be due for instance to a change in interest rates, which implies an increase of default of mortgages (as in the recent subprime crisis), to the default of an institution out of the perimeter of interest, etc.

(ii) The reduced form approach tries to identify the two components of risk dependencies without the structural knowledge of the interconnection between the individual risks. The reduced form models for contagion consider specifications that capture how the defaults of some individuals influence the default intensities of the individuals still alive. They are either (a) dynamic models written in continuous time, at the individual level,[2] and based on the notion of mutually exciting point processes introduced by Hawkes (1971a,b) and Hawkes and Oakes (1974), or (b) specifications based on the epidemic model introduced by Bailey (1953, 1957) and Kendall (1956) (see the so-called infectious model used in a static framework by Davis and Lo, 2001a,b; Sakata et al., 2007, and its dynamic extension by Rulliere et al., 2013). Specifications including both frailty and contagion effects are introduced e.g. in Frey and Backhaus (2003), Giesecke and Weber (2004, 2006), Azizpour and Giesecke (2008), Azizpour et al. (2011), and Lando and Nielsen (2010). For the analysis of contagion and frailty effects, it is important to consider rather large portfolios of individual risks, which are sufficiently homogeneous with respect not only to their marginal risk distribution, but also to the two types of dependencies. Indeed, the Basel 2 regulation has already considered such benchmark models for the analysis of large portfolios of loans, mortgages and Credit Default Swaps when risk dependence arises from frailty effects only (see e.g. Schoenbucher, 2001; Frey and McNeil, 2003). For such models, portfolio risk measures can be analytically approximated by their cross-sectional (i.e. large portfolio) limits (Vasicek, 1987), including first-order corrections called granularity adjustments (Gordy, 2003, 2004; Gagliardini et al., 2012; Gagliardini and Gourieroux, forthcoming). This type of aggregation approach is not only useful for retail financial products, but can also be used to gather the small and medium size institutions in a banking system, not yet enough concentrated. The aim of this paper is to develop a similar approach of aggregation of individual risks, taking into account not only the frailty effects, but also the contagion effects, in order to analyse the risk of the system.

In our paper, we consider this problem for a transition model in discrete time, at a semi-aggregate level with respect to time and individual. Instead of following the dynamics of individual risks, we focus on the dynamics of counts of individuals in the different classes of risk. Therefore, we introduce dynamic models for count processes, with both systematic factors and contagion. The analysis at a semi-aggregate level has several advantages. First, it is less demanding in terms of data confidentiality. Second, it allows for a modeling by means of affine processes, which are tractable for prediction purposes and for studying the stationarity properties. Third, the dynamics at the semi-aggregate level are sufficient for risk forecasting, e.g. computing portfolio risk measures, and pricing credit derivative assets whose payoff is a function of the portfolio loss, such as Collateralized Debt Obligations (CDO).

[1] The links between balance sheets are much more important for financial institutions than for firms of other industrial sectors (see e.g. Benmelech and Bergman, 2011, for a discussion of contagion in the airline industry).

[2] These models have to be distinguished from dynamic macroscopic models introduced to capture the volatility transmission (Gallo and Otranto, 2007), or the jump transmission (Ait-Sahalia et al., 2010) between markets, even if they share common features with the microscopic models introduced for individual risks.