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Non-smooth dynamics and multiple equilibria in a Cournot–Ramsey model with endogenous markups

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ABSTRACT

We consider a Ramsey model with a continuum of Cournotian industries where free entry generates an endogenous markup. The model produces two different regimes, monopolistic and Cournotian monopolistic competition, resulting in non-smooth dynamics. We analyze the global dynamics of the model, demonstrating it may exhibit heteroclinic orbits connecting multiple equilibria. Small transitory changes in parameters can lead to large permanent effects and there can be a poverty trap separating a low-capital and high-markup equilibrium from a high-capital low-markup equilibrium. We apply results from the mathematics of non-smooth dynamic systems, which provide a more general framework for understanding regime switching.

1. Introduction

In this paper we develop a dynamic general-equilibrium continuous-time Ramsey model with endogenous entry in Cournot product markets, resulting in an endogenous markup. Cournot competition has the attraction that increases in economic activity are associated with more firms and hence lower markups, which captures the empirical feature of counter-cyclical markups–see *inter alia* (Martins and Scarpetta, 2002). However, we show that the natural *lower bound* of one firm per industry introduces the possibility of multiple equilibria and a switch in dynamics. To analyze the Cournot model with endogenous entry, we need to develop an approach which allows for more than one steady state and different dynamic regimes around each steady state, separated by a *switching frontier*: a *Cournotian Monopolistic Competition* (CMC)[1] regime in which there is more than one firm per industry, and higher output and capital lead to lower counter-cyclical markups; and a *Monopolistic Competition* (MC) regime, in which there is one firm per industry and lower output and capital lead to a larger acyclic markup.

Whilst this is of theoretical interest, we believe that it may provide a more general framework for understanding how economies may move from one type of dynamic behavior to another. We show that there can be up to three equilibria in this economy: one when the capital stock is low which is monopolistic and saddle-point stable, and up to two CMC

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[1] The term Cournotian Monopolistic Competition was coined by D'Aspremont et al. (1991). It refers to the case where the firms are Cournot price-makers in their own industry, but take the aggregate economy-wide price as given (as in monopolistic industries).

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equilibria only one of which can be saddle-point stable, the other (if it exists) being a source. The intuition for the three equilibria is that the marginal revenue product (MRP) may be non-monotonic in capital: for low levels of capital with one firm per industry, the MRP of capital is decreasing; in the CMC regime, entry leads to a falling markup as capital increases, and this may produce an increasing MRP function for a range, before eventually the diminishing marginal product of capital dominates. This non-monotonicity in the MRP of capital means that it can be equated to the rate of time preference at one, two or three different levels of the capital stock. This means that the economy can have up to two stable states: a high markup equilibrium with low output, and a low markup equilibrium with high output.[2]

The economic implications of the existence of multiple equilibria and regime switches are quite profound. Two fundamentally similar economies may behave very differently, as they may be in two different regimes with distinct dynamic behavior, especially in terms of markups. Even for the same economy, there is the possibility of regime change along the convergence to a stable long-run equilibrium if the switching boundary is crossed. From the bifurcation analysis, the "*deep parameters*" associated with the dominant market structure (fixed costs and the elasticity of demand) play a crucial role in this model and a change in their values may alter the dynamics in a radical way, either by inducing a discontinuous transition or a discontinuous hysteresis. Even a *transitory* technology shock can give rise to a *large permanent* shift in the equilibrium if it pushes the economy through the switching boundary to the alternative stable equilibrium. The initial shock can be amplified through capital accumulation as it passes through the switching boundary. Standard models with a unique regime and steady state imply that any shock will lead back to the initial steady state.

Whilst there are several papers that have explored the relationship between entry and the resultant "endogenous markups" using the Cournot model, they are in a different framework to ours. Most have done so in discrete-time either in an overlapping-generations environment, as in Chatterjee et al. (1993), D'Aspremont et al. (1996), dos Santos Ferreira and Lloyd-Braga (2005) or Kaas and Madden (2005), or in a unique steady-state Real Business Cycle setting as in dos Santos Ferreira and Dufourt (2006), Portier (1995), Costa (2001, 2006). Also, somewhat related but different continuous-time models of endogenous growth have employed the Cournot mechanism for endogenizing markups (Zilibotti, 1994; Galí and Zilibotti, 1995). In contrast to the Cournotian approach of this paper is the Linnemann (2001) model of entry in monopolistic competition as used in Jaimovich (2007) and Jaimovich and Floetotto (2008), and Bilbiie et al. (2012). Entry reduces the market share of firms, and hence reduces the "own-price effect" of the monopolistic producer on the aggregate price index, which increases the elasticity of demand (see Yang and Heijdra, 1993).[3] However, the papers using this "own-price effect" do so with only a unique equilibrium, so that the issues we explore are not present. The contribution of our paper is to provide a comprehensive analysis of the local and global dynamics in a model with not only multiple equilibria, but also in which there are two dynamic regimes. We apply recent advances in applied mathematics (di Bernardo et al., 2008a,b) which extend traditional analysis to allow for piecewise-smooth (PWS) dynamic systems: at the switching frontier, the functions defining the dynamics may be continuous but non-differentiable.[4]

There is an empirical evidence of a negative cross-country relationship between the markup and wealth. Galí (1995) used the labor share in total income as a proxy for the markup and found a negative relationship across a wide range of 138 countries. If we restrict ourselves to *developed* economies, we find a similar relationship using the data in Martins et al. (1996).[5] The model also predicts that the volatility of markups will be different: higher income countries will have low and variable markups and lower income countries constant markups. The predictions on volatility are not practically testable, since other factors may affect markups (e.g. nominal rigidity, productivity shocks) and most developed economies have not experienced radical shifts in labor's share. However, Greece and Portugal experienced structural changes that we may identify with long-run regime shifts: both had a large fall in labor's share associated with a significant reduction in volatility.[6]

This paper is organized as follows. In Section 2, we introduce the basic Ramsey model with capital accumulation and entry *à la* Cournot. In Section 3, we define general equilibrium and its representation by a piecewise-smooth dynamic system. In Section 4 we characterize local dynamics in the neighborhoods of the stationary equilibria and of the switching frontier. In Section 5, the typology of global dynamics is presented. Section 6 analyzes the long-run effects of technology shocks leading to permanent regime changes, Section 7 concludes.

[2] Whilst the current model does not explicitly deal with growth or development, there is an obvious parallel with the "Rostovian" threshold effect, or "poverty trap" (Easterly, 2006).

[3] Other papers that consider a variety of aggregate feedback mechanisms are D'Aspremont et al. (1989) and Wu and Zhang (2000).

[4] There are other applications of non-smooth dynamics in economic theory, without an explicit reference to the relevant mathematical literature as Matsuyama (2004) or Peretto (2007). In the mathematics literature Laura Gardini and co-workers have an extensive number of applications to economics, e.g. Tramontana et al. (2011).

[5] We used the smaller, but more reliable sample in Martins et al. (1996) that provided production-weighted average markups for 14 OECD countries based on 36 manufacturing industries in the 1970–1992 period. The correlation of these markup measures with 1970 real GDP per capita (PPP) was –0.48 and with capital stock per capita it was –0.53, using the data in the AMECO database. Details available from the authors on request.

[6] We compare them with a group of 17 industrialized countries for which there is a continuous labor-share time series in AMECO from 1960 to 2010. Data is available from the authors on request.