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Topology optimization of thermo-elastic structures with multiple materials under mass constraint

Gao Tong *, Xu Pengli, Zhang Weihong

Engineering Simulation and Aerospace Computing (ESAC), School of Mechanical Engineering, Northwestern Polytechnical University, 710072 Xi'an, China

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This work addresses the complicated design problem in which a structure of multiple materials is topologically optimized under the conditions of steady-state temperature and mechanical loading. First, the general thermal stress coefficient (GTSC) is introduced to relate the thermal stress load to the design variables and address an engineering practice need by breaking down the previous assumption that the Poisson's ratios of all candidate materials are the same. Second, the Uniform Multiphase Materials Interpolation (UMMI) scheme and the Rational Approximation of Material Properties (RAMP) scheme are combined to parameterize material properties (e.g., the elasticity matrix and GTSC). In the problem formulation, mass constraint is adopted to automatically determine the optimal match of candidate materials instead of imposing the standard volume constraint to each material phase. An improved optimization formulation with an artificial penalty term is also proposed to avoid a possible mixed material status in the numerical results. Numerical tests illustrate the validity of the proposed method.

1. Introduction

In recent years, structural topology optimization has been recognized to be a challenging research topic in the engineering design community. Standard compliance minimization with a single material phase and pure mechanical loads was extended to various complicated design problems to achieve innovative structure layouts. Recent developments in topology optimization were reviewed in Refs. [1–3].

Topology optimization methods can generally be classified into two types according to how the design variables are defined. The first type involves the density method in which each design variable is associated with a finite element and describes the presence (1) or absence (0) of the solid material. To do this, a variety of schemes have been developed to transform the original 0–1 discrete optimization problem into a continuous one, such as the homogenization method [4], SIMP (Solid Isotropic Material with Penalization) scheme [5], RAMP (Rational Approximation of Material Properties) scheme [6] and ESO (Evolutionary Structural Optimization) method [7]. The second type handles topology optimization as a generalized shape optimization problem with design variables associated with the structural boundary. The level set method [8] and the similar phase field method [9] are typical examples.

Continuum topology optimization with multiphase materials was first investigated by Thomsen [10]. Later, typical works focused on the extension of the SIMP/RAMP schemes and handled varieties of topology optimization problems with multiple materials that include: the design of micro-structures with the extreme equivalent property [11], thermo-elastic problem subjected to the volume constraint [12] and multi-physics actuator design [13]. Simultaneous design of the structural layout and discrete fiber orientation was also dealt with using an extension of the SIMP scheme, for example, the so-called DMO (Discrete Material Optimization) scheme [14], SFP (Shape Functions with Penalization) scheme [15] and BCP scheme (Bi-value Coding Parameterization) [16]. Meanwhile, the ESO was also applied to address multiple materials [17]. Similarly, an evolutionary approach using discrete variables was proposed to solve the mass minimization problem with multiple materials and strength constraints [18]. Alternatively, the level set method and the phase field method were applied to address multi-material topology optimization problems, including both the stiffness maximization problem [19] and heat conduction problem [20]. It should be noted that the implicit description of the interfaces between two distinct solid material phases is the basis of this approach. Some other schemes should be mentioned here: Yin and Ananthasuresh [21] proposed a

* Corresponding author.
E-mail address: gaotong@nwpu.edu.cn (T. Gao).

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multi-material interpolation model based on the so-called peak function; Jung and Gea [22] constructed a variable-inseparable multi-material model for the design of an energy-absorbing structure; and Yoon [23] presented the so-called patch stacking method for the nonlinear dynamic problem with multiple materials.

However, in most works, the material amount was controlled by the volume constraint of each candidate phase. In the engineering design sense, the volume constraint is less significant than the mass constraint of the whole structure. Although both constraints are identical when only one single solid material phase is present, the situation changes completely in the case of multiple materials due to the differences in material densities. In previous work, the mass constraint of multiple materials was investigated only for the case using pure mechanical loads for the structural compliance minimization [24]. Two interpolation schemes, namely, RMMI (Recursive Multiphase Materials Interpolation) and UMMI (Uniform Multiphase Materials Interpolation), were discussed and compared. It was also demonstrated that the mass constraint is more beneficial than the volume constraint in the sense that the former can further increase structural stiffness and automatically match multiple material properties (i.e., Young's modulus and density for the same amount of structure mass).

Comparatively, topology optimization of thermal structures is more complicated because it belongs to a type of design-dependent problem [25], with the thermal stress load changing with the spatial distribution of solid material phases. Rodrigues and Fernandes [26] adopted the homogenization method to formulate the thermal stress load for the mean compliance minimization. Li et al. [27] used the ESO method with element thickness to be design variables. An adjoint design sensitivity analysis method [28] was developed for the topology optimization of weakly coupled thermo-elastic problems. Structural rigidity optimization with an initial design-dependent thermo-elastic stress field was also presented [29]. Deng et al. [30] optimized the microstructure of homogeneous porous material and macrostructure topology. Pedersen and Pedersen [31] found that the minimization of the maximum von-Mises stress could be achieved by applying a procedure to obtain the uniform energy density. Recent results from Zhang et al. [32] indicated that the elastic strain energy minimization and mean compliance minimization led to different configurations if thermal loads exist. The elastic strain energy minimization particularly favors stress reduction. Multiple materials were taken into account [12]. The concept of the thermal stress coefficient (TSC) was introduced as the product between Young's modulus and the thermal expansion coefficient based on the assumption that Poisson's ratios of all candidate materials are the same. The TSC was adopted later in the thermo-elastic topology optimization of stress-constrained problems [33] and dynamic compliance minimization [34].

This work focuses on a topology optimization with multiple materials under the mass constraint and the conditions of steady-state temperature and mechanical loading. This paper is organized as follows. In Section 2, the GTSC is introduced to relate the thermal stress load to the design variables. In Section 3, the UMMI and RAMP schemes are combined to parameterize the properties of multiple materials. In Section 4, the standard optimization formulation of thermo-elastic structures and a sensitivity analysis is first presented. Then, the formulation of the mass constraint is presented and theoretically compared with the volume constraint. Finally, the mixed material status by means of the standard optimization formulation is illustrated and an improved optimization formulation is proposed by the introduction of an artificial penalty term with a variable parameter. In Section 5, numerical examples illustrate the validity of the proposed optimization method. In the last section, the conclusions and contributions are presented.

2. General thermal stress coefficient

In this work, it is assumed that the material properties are temperature-independent, and only the steady-state temperature field is taken into account. As is well known for a finite element model, the nodal vector of the thermal stress load of the ith element is expressed as

$$\mathbf{F}_i^{\text{th}} = \int_{\Omega_i} \mathbf{B}_i^{\text{T}} \mathbf{D}_i \boldsymbol{\varepsilon}_i^{\text{th}} d\Omega \tag{1}$$

Here, $\mathbf{B}_i$ is element strain–displacement matrix consisting of element shape function derivatives and is independent of the topology design variables. $\mathbf{D}_i$ is the parameterized elasticity matrix dependent on Young's modulus E_i and Poisson's ratio μ_i. Previously, $\mathbf{D}_i$ was expressed as a function of Young's modulus and Poisson's ratio was assumed to be constant [12]. In this work, isotropic linear elastic materials are used, and the elasticity matrix $\mathbf{D}_i$ for 2D and 3D problems can be written as

$$\mathbf{D}_i = \frac{E_i(1-\mu_i)}{(1+\mu_i)(1-2\mu_i)} \begin{bmatrix} 1 & \frac{\mu_i}{1-\mu_i} & 0 \\ \frac{\mu_i}{1-\mu_i} & 1 & 0 \\ 0 & 0 & \frac{1-2\mu_i}{2(1-\mu_i)} \end{bmatrix} \quad \text{for 2D problems}$$

$$\mathbf{D}_i = \frac{E_i(1-\mu_i)}{(1+\mu_i)(1-2\mu_i)} \begin{bmatrix} 1 & \frac{\mu_i}{1-\mu_i} & \frac{\mu_i}{1-\mu_i} & 0 & 0 & 0 \\ \frac{\mu_i}{1-\mu_i} & 1 & \frac{\mu_i}{1-\mu_i} & 0 & 0 & 0 \\ \frac{\mu_i}{1-\mu_i} & \frac{\mu_i}{1-\mu_i} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\mu_i}{2(1-\mu_i)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\mu_i}{2(1-\mu_i)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\mu_i}{2(1-\mu_i)} \end{bmatrix} \quad \text{for 3D problems} \tag{2}$$

The thermal strain vector $\boldsymbol{\varepsilon}_i^{\text{th}}$ is written as

$$\boldsymbol{\varepsilon}_i^{\text{th}} = \boldsymbol{\alpha}_i \cdot \Delta \mathbf{t}_i \tag{3}$$

where $\Delta \mathbf{t}_i$ denotes the temperature rise vector of the ith element. The thermal expansion coefficient vector, $\boldsymbol{\alpha}_i$, can be written as

$$\begin{aligned} &\boldsymbol{\alpha}_i = \alpha_i \boldsymbol{\phi} \\ &\boldsymbol{\phi} = [1 \quad 1 \quad 0] \quad \text{for 2D problems} \\ &\boldsymbol{\phi} = [1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0] \quad \text{for 3D problems} \end{aligned} \tag{4}$$

where α_i is the thermal expansion coefficient of element i.

The substitution of $\boldsymbol{\varepsilon}_i^{\text{th}}$ into Eq. (1) then produces

$$\mathbf{F}_i^{\text{th}} = \int_{\Omega_i} \mathbf{B}_i^{\text{T}} \mathbf{D}_i \boldsymbol{\alpha}_i \Delta \mathbf{t}_i d\Omega \tag{5}$$

Then, the thermal stress coefficient vector is defined as:

$$\boldsymbol{\beta}_i = \mathbf{D}_i \boldsymbol{\alpha}_i \tag{6}$$

The substitution of Eqs. (2) and (4) into Eq. (6) yields

$$\boldsymbol{\beta}_i = \frac{E_i \alpha_i}{1-2\mu_i} \boldsymbol{\phi} \tag{7}$$

in which the GTSC can be defined below as

$$\beta_i = \frac{E_i \alpha_i}{1-2\mu_i} \tag{8}$$

and can be treated as an inherent material property. For comparison, in a previous work [12], the TSC was expressed as a function of Young's modulus and the thermal expansion coefficient because Poisson's ratios of all of the candidate materials were supposed the same.