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Bayesian analysis of structural credit risk models with microstructure noises

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ABSTRACT

In this paper a Markov chain Monte Carlo (MCMC) technique is developed for the Bayesian analysis of structural credit risk models with microstructure noises. The technique is based on the general Bayesian approach with posterior computations performed by Gibbs sampling. Simulations from the Markov chain, whose stationary distribution converges to the posterior distribution, enable exact finite sample inferences of model parameters. The exact inferences can easily be extended to latent state variables and any nonlinear transformation of state variables and parameters, facilitating practical credit risk applications. In addition, the comparison of alternative models can be based on deviance information criterion (DIC) which is straightforwardly obtained from the MCMC output. The method is implemented on the basic structural credit risk model with pure microstructure noises and some more general specifications using daily equity data from US and emerging markets. We find empirical evidence that microstructure noises are positively correlated with the firm values in emerging markets.

1. Introduction

Credit risk is referred to as the risk of loss when a debtor does not fulfill its debt contract and is of natural interest to practitioners in the financial industry as well as to regulators. For example, it is common practice that banks use securitization to transfer credit risk from bank's balance sheets to the market. The credit problem can well become a crisis when some of the risk lands back on banks. The turbulence in international credit markets and stock markets at the end of 2007 has largely been caused by this subprime credit problem in the US. To a certain degree, the 1997 Asian financial crisis was also caused by this credit risk problem. Not surprisingly, how to the credit risk is assessed is essential for risk management and for the supervisory evaluation of the vulnerability of lender institutions. Indeed, the Basel Committee on Banking Supervision has decided to introduce a new capital adequacy framework which encourages the active involvement of banks in measuring the likelihood of defaults. The growing need for the accurate assessment of credit risk motivates academicians and practitioners to introduce theoretical models for credit risk.

A widely used approach to credit risk modelling in practice and also in the academic arena is the so-called structural method. This method of credit risk assessment was first introduced by Black and Scholes (1973) and Merton (1974). In this approach the dynamic behavior of the value of a firm's assets is specified. If the value becomes lower than a threshold which is usually a proportion of the firm's debt value, the company is considered to be in default. For example, in Black and

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Scholes (1973) and Merton (1974), a simple diffusion process is assumed for a firm's asset value, and the firm will default if its asset value is lower than its debt on the maturity date of the debt.

Since the firm's asset value is not directly observed by econometricians, the econometric estimation of structural credit risk models is nontrivial. To deal with the problem of unobservability, Duan (1994) introduces a transformed data maximum likelihood (ML) method, using observed time series data on publicly traded equity values. The idea essentially is to use the change-of-variable technique via the Jacobian, relying critically on the one-to-one correspondence between the traded equity value and the unobserved firm's asset value. Since then, this method has been applied in a number of studies; see for example, Wong and Choi (2006), Ericsson and Reneby (2004) and Duan et al. (2003). Duan et al. (2004) showed that the method is equivalent to the Moody's KMV model, a popular commercial product.

It is well known in the market microstructure literature that the presence of various market microstructure effects (such as price discreteness, infrequent trading and bid–ask bounce effects) contaminates the efficient price process with noises. There have been extensive studies on analyzing the time series properties of microstructure noises. Some earlier contributions include Roll (1984) and Hasbrouck (1993). In recent years, various specifications have been suggested for modelling microstructure noise in ultra-high frequency data in the context of measuring daily integrated volatility. Examples include the pure noise (i.e. iid) model (Zhang et al., 2005; Bandi and Russell, 2008), stationary models (Aït-Sahalia et al., 2009; Hansen and Lunde, 2006) and locally nonstationary models (Phillips and Yu, 2006, 2007). The consensus emerging from the literature is that if the microstructure noise were ignored, one would get an inconsistent estimate of the quantity of interest. This implication is also confirmed in Duan and Fulop (2009) in the context of credit risk modelling.

However, if the observed equity prices are contaminated with microstructure noises in structure credit risk models, the one-to-one correspondence between the traded equity value and the unobserved firm's asset value is broken, and hence the method developed in Duan (1994) is not applicable anymore. A fundamental difficulty is that neither the efficient prices nor microstructure noises are observable. As a result, the change-of-variable technique becomes infeasible. In an important contribution, Duan and Fulop (2009) developed a simulation-based ML method to estimate the Merton model with Gaussian iid microstructure noises. The ML method is designed to deal with nonlinear non-Gaussian state space models via particle filtering. In the credit risk model with microstructure noises, the nonlinear relationship between the contaminated traded equity value and firm's asset value is given by the option pricing model but is perturbed by microstructure noises. This gives the observation equation. The state equation specifies the dynamics of the asset value in continuous time, usually with a unit root.

The standard asymptotic theory for the ML estimator, such as asymptotic normality and asymptotic efficiency, is then called upon to make statistical inferences about the model parameters and model specifications. Most credit risk applications require the computation of nonlinear transformation of model parameters and the unobserved firm's asset value. The invariance principle is employed for obtaining the ML estimates of these quantities. The delta method is utilized to obtain the asymptotic normality and to make statistical inferences asymptotically. Duan and Fulop (2009) followed this tradition. Using simulations, Duan and Fulop checked the reliability of the standard asymptotic theory. Their results indicate that the asymptotic theory does not work well for the trading noise parameter while ML provides accurate estimates.

One reason for the departure of the finite sample distribution from the asymptotic distribution is the boundary problem. This reason has been put forward by Duan and Fulop and effectively demonstrated via Monte Carlo simulations. We believe, however, there is another reason for the departure. If the microstructure noise process is stationary, the model represents a parametric nonlinear cointegrated relationship between the observed equity value and the unobserved firm's asset value. Park and Phillips (2001) showed that in nonlinear regressions with integrated time series, the limiting distribution is nonstandard and the rate of convergence depends on the properties of nonlinear regression function. As a result, the standard asymptotic theory for ML, such as asymptotic normality, may not be valid.

The first contribution of this paper is to introduce an alternative likelihood-based inferential method for Merton's credit risk model with iid microstructure noises. The new method is based on the general Bayesian approach with posterior computations performed by Gibbs sampling, coupled with data augmentation. Simulations from the Markov chain whose stationary distribution converges to the posterior distribution enable exact finite sample inferences. We note that Jacquier et al. (1994) and Kim et al. (1998), among others, have suggested this approach in the context of a stochastic volatility model. We recently became aware that this idea has independently been discussed by Korteweg and Polson (2009) in the context of Merton's credit risk model with iid microstructure noises.[1]

There are certain advantages in the proposed method. First, as a likelihood-based method, MCMC matches the efficiency of ML. Second, as a by-product of parameter estimation, MCMC provides smoothed estimates of latent variables because it augments the parameter space by including the latent variables. Third, unlike the frequentist's methods whose inference is almost always based on asymptotic arguments, inferences via MCMC are based on the exact posterior distribution.

[1] Our work differs from this paper in several important respects. First, while we adopt the specification of the state equation of Duan and Fulop (2009) by perturbing the log-price with an additive error, Korteweg and Polson (2009) assume a multiplication error on the state variable and require the pricing function be invertible. Second, our work goes beyond the estimation problem to encompass issues involving model comparisons. Third, we examine more flexible microstructure noise behavior based on stock prices only whereas Korteweg and Polson (2009) used the pure noise normality assumption based on multiple price relations.