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Solving the multi-country real business cycle model using a Smolyak-collocation method

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ABSTRACT

We describe a sparse-grid collocation method to compute recursive solutions of dynamic economies with a sizable number of state variables. We show how powerful this method can be in applications by computing the non-linear recursive solution of an international real business cycle model with a substantial number of countries, complete insurance markets and frictions that impede frictionless international capital flows. In this economy, the aggregate state vector includes the distribution of world capital across different countries as well as the exogenous country-specific technology shocks. We use the algorithm to efficiently solve models with up to 10 countries (i.e., up to 20 continuous-valued state variables).

1. Introduction

In this paper we propose a projection method based on Smolyak's (1963) algorithm to compute *globally accurate* solutions to models characterized by a sizeable number of continuous-valued state variables, such as international real business cycle models with a substantial number of countries. The use of a sparse grid constructed with Smolyak's algorithm allows us to handle (at least) 20 state variables, while solving the model with this many state variables is not possible when standard full grids are used.

Our objectives are twofold. First, we aim at providing an easily accessible general description of our algorithm, replacing and improving upon Krueger and Kubler (2004). Second, we show how powerful this method is by numerically solving an international real business cycle model with many countries and international capital market frictions.[1] The introductory articles to this issue, Denhaan et al. (this issue) and Juillard and Villemont (this issue), together provide a full description of the international real business cycle model that is to be solved. We therefore only repeat it here insofar as is needed for the description of the algorithm. Furthermore, Juillard and Villemont (this issue) describe the accuracy tests with which our and competing solution methods are evaluated, while the paper by Kollmann et al. (this issue-b) compares the

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[1] For a description and application of this class of models, see e.g. Backus et al. (1992, 1995).

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performance of our algorithm relative to these competing methods.[2] We therefore defer the detailed discussion of the performance of the algorithm to the latter paper. To summarize the main findings, our sparse grid projection method performs quite well for a wide variety of model specifications including models with up to 10 countries (i.e., 20 continuous-valued state variables), specifications that introduce a great deal of curvature into utility and production functions, and models with asymmetries between countries. Our method is also substantially more accurate than a linear approximation of the solution, especially when the exogenous shocks to the economy are large.

Section 2 provides a general description of our projection method, and Section 3 discusses key implementation details. The final section offers some short concluding remarks.

2. A sparse-grid collocation method

The model we solve, to demonstrate the scope as well as the advantages and shortcomings of our method, is the international real business cycle model with N countries and capital adjustment costs.

2.1. The application

Due to adjustment costs, the state variables for the recursive formulation of the social planner's problem consist of the vector of exogenous current productivity levels $a=(a^1,\ldots,a^N)$ and the vector of endogenous current capital stocks $k=(k^1,\ldots,k^N)$. Denote by $s=(k,a)$ the current state, which is of dimension $2N$. We defer a complete description of the model, the interpretation of the functional forms and their parameterization to the introductory article, Juillard and Villemont (this issue), and in this section only develop the notation needed to describe the application of the Smolyak algorithm to this model.

The planner's problem can be written recursively as

$$V(k,a)=\max_{\{c^j,l^j,k'j\}}\sum_{j=1}^{N}\tau^j u^j(c^j,l^j)+\beta\int V(k',a')g_a(a')\,da' \tag{1}$$

s.t.

$$\ln(a'j)=\rho\ln(a^j)+\sigma(e'j+e') \tag{2}$$

$$\sum_{j=1}^{N}\left(c^j+k'j+\frac{\phi}{2}\frac{(k'j-k^j)^2}{k^j}\right)=\sum_{j=1}^{N}(a^j f^j(k^j,l^j)+k^j) \tag{3}$$

where $g_a(a')$ denotes the probability density function over a', given a. We will now derive the system of functional equations used to compute this model. We seek functions $C^j(s)$, $L^j(s)$, and $K'j(s)$ for $j=1,\ldots,N$, mapping the current state $s=(k,a)$ into consumption and labor supply of each country today and its capital stock tomorrow. For future reference we define

$$C(s)=\sum_{j=1}^{N}C^j(s) \tag{4}$$

$$Y(s)=\sum_{j=1}^{N}a^j f^j(k^j,L^j(s)) \tag{5}$$

$$K=\sum_{j=1}^{N}k^j \tag{6}$$

$$K'(s)=(K'1(s),\ldots,K'N(s)) \tag{7}$$

Attaching Lagrange multiplier λ to the resource constraint, we find as first-order conditions

$$\tau^j u_c^j(c^j,l^j)=\lambda \quad \forall j \tag{8}$$

$$\frac{\tau^j u_l^j(c^j,l^j)}{-a^j f_l^j(k^j,l^j)}=\lambda \quad \forall j \tag{9}$$

$$\frac{\beta\int V_{k^j}(k',a')g_a(a')\,da'}{1+\frac{\phi(k'j-k^j)}{k^j}}=\lambda \quad \forall j \tag{10}$$

[2] Alternative algorithms described in this issue include those developed by Kollmann et al. (this issue-a), Maliar et al. (this issue), and Pichler (this issue).