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Spatial discounting, Fourier, and racetrack economy: A recipe for the analysis of spatial agglomeration models ☆

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ABSTRACT

We provide an analytical approach that facilitates understanding the bifurcation mechanism of a wide class of economic models involving spatial agglomeration of economic activities. The proposed method overcomes the limitations of the Turing (1952) approach that has been used to analyze the emergence of agglomeration in the multi-regional core–periphery (CP) model of Krugman (1993, 1996). In other words, the proposed method allows us to examine whether agglomeration of mobile factors emerges from a uniform distribution and to analytically trace the evolution of spatial agglomeration patterns (i.e., bifurcations from various polycentric patterns as well as a uniform pattern) that these models exhibit when the values of some structural parameters change steadily. Applying the proposed method to a multi-regional CP model, we uncover a number of previously unknown properties of the CP model, and notably, the occurrence of "spatial period doubling bifurcation" in the CP model is proved.

1. Introduction

Most of the world's population and economic activities are strikingly concentrated in a limited number of areas. Although thousands of articles have been devoted to explain the prevalence of this pattern, the *core–periphery* (CP) *model* developed by Krugman (1991) is the first successful attempt to explain this in a full-fledged general equilibrium framework (Fujita and Thisse, 2009). The CP model introduces the Dixit and Stiglitz (1977) model of monopolistic competition and increasing returns into spatial economics and provides a basic framework by which to describe how the interaction among increasing returns, transportation cost, and factor mobility can cause the formation of economic

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agglomeration in geographical space. These new modeling techniques further led to the development of a new branch of spatial economics, known as "*new economic geography* (NEG)", which has become a fast-growing field. Over the past few decades, numerous extensions of the original CP model have appeared,[1] and notably, there has been a proliferation of theoretical and empirical research applying the NEG theory framework to deal with various policy issues, such as trade policy, taxation, or regional redistribution in recent years (Baldwin et al., 2003; Combes et al., 2008).

As indicated by its remarkable expansion, the NEG theory has the potential to provide a solid basis for predicting and evaluating the long-run effects of various economic policy proposals. There remain, however, some fundamental limitations that must be addressed before the NEG theory can be confirmed to provide a sound foundation for such applications. One of the most relevant issues is that the theoretical results are mainly restricted to two-region cases. Even though some efforts to overcome this limitation have been made in the early stages of development of the NEG (e.g., Krugman, 1993, 1996; Fujita et al., 1999b), theoretical studies on the NEG models have focused almost exclusively on two regions in the last decade.

Although clearly the correct starting point for analysis, two-region models have several limitations. First, these models can neither describe nor explain a rich variety of *polycentric patterns* that are prevalent in real-world economies. Second, these models cannot provide an adequate framework to consider spatial interactions in a well-defined sense. For example, consider the concept of market accessibility, the differences among regions of which, as evidenced by numerous empirical studies, are major factors that lead to various forms of spatial agglomeration. Note that this concept is defined as the relative position of each region *within the entire network of interactions*. Due to the existence of such network effects, the impacts of a change in market accessibility in a multi-regional system are quite different from those in a two-region system, the topology of which is too simple to capture the network effects (Behrens and Thisse, 2007).

In view of the limitations of two-region models, it remains unclear as to whether we can extrapolate the predictions and implications derived from two-regional analysis to a multi-regional system (Behrens and Thisse, 2007). Therefore, it is fair to say that "*real-world geographical issues cannot be easily mapped into two-regional analysis*" (Fujita et al., 1999b, p. 79), and the multi-regional framework is a prerequisite for systematic empirical research as well as for practical applications of the NEG theory.

In the present paper, an attempt is made to clarify the spatial agglomeration properties of a *multi-regional CP model* in which regions are located on a circumference. As described in the NEG literature, the two-region CP model exhibits a bifurcation from a symmetric equilibrium (spatially uniform distribution of population) to an asymmetric equilibrium (agglomeration of population), depending on the values of certain structural parameters. In analyzing the multi-regional CP model, the most important, yet most difficult, step is examining the more complex bifurcation (i.e., *evolution of spatial agglomeration structure*). In the present paper, we investigate the mechanism of the bifurcation *through a novel analytical approach* synthesizing several techniques each of which is available in economics or in nonlinear mathematics. We shall explain this approach later in this section. For the analysis, we first present a multi-regional version of Pflüger's (2004) two-regional CP model, which is a slight modification of Krugman's original CP model, so that short-run equilibrium is obtained analytically.[2] We then show a complete picture of the *evolutionary processes of agglomeration patterns* in the CP model with K regions equidistantly located on a circumference. The term evolutionary process means that we consider a process in which transportation cost parameter τ steadily decreases over time, starting from a very high value of τ, at which a uniform distribution of firms/workers is a stable equilibrium.

It is first shown that the critical value of τ at which the agglomeration structure first emerges (i.e., the first bifurcation occurs) and the associated agglomeration pattern can be obtained analytically. While the first bifurcation is the final and only bifurcation in conventional two-region models, the multi-regional CP model does not end the story. It is indeed proved that the *spatial configuration of population evolves in association with further decreases in* τ, accompanied by the second bifurcation. The analysis of these two bifurcations further reveals that they share a common property, namely the number of core regions (in which firms/workers agglomerate) is reduced by half and the spacing between each pair of neighboring core-regions (*agglomeration shadow*) doubles after each bifurcation. This suggests a general rule whereby the multi-regional CP economy exhibits the *spatial period doubling bifurcation* (SPDB). Finally, a proof of this conjecture, including a general analytical formula to predict the occurrence of the SPDB, is presented.[3] In addition to these results, we further uncover the effect of the heterogeneity of consumers on agglomeration patterns. Specifically, it is deduced that the CP model may exhibit repeated agglomeration and dispersion with the steady decrease in τ, which provides a generalization, as well as a theoretical explanation, of several *bell-shaped development* results reported in studies on two-region CP models.[4]

[1] For comprehensive reviews of the literature on the NEG/CP models, see the following monographs and handbooks: Fujita et al. (1999b), Papageorgiou and Pines (1998), Fujita and Thisse (2002), Baldwin et al. (2003), Henderson and Thisse (2004), Combes et al. (2008), Glaeser (2008), and Brakman et al. (2009).

[2] The approach presented herein, in principle, is applicable to Krugman's (1991) original CP model and its variants (e.g., Ottaviano et al., 2002; Forslid and Ottaviano, 2003), but we chose this Pflüger's model, in favor of its analytical solvability, which is vital in the clarity of exposition.

[3] While Krugman (1993, 1996) has presented some numerical results that can be regarded as special examples of the SPDB, no study has presented a rigorous proof of the general SPDB in the CP model.

[4] See Tabuchi (1998), Helpman (1998), and Murata (2003).