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Linear rational-expectations models with lagged expectations: A synthetic method ☆

Alexander Meyer-Gohde *

Technische Universität Berlin, Sek. H 52, Straße des 17. Juni 135, 10623 Berlin, Germany

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ABSTRACT

This paper contains a solution and an estimation method for linear rational-expectations models with lagged expectations. The solution method is a synthetic approach, combining state-space and infinite-MA representations with a simple system of linear equations. The advantage lies in the particular combination of methods from the literature, providing faster execution, more general applicability, and more straightforward usage than existing algorithms. Bayesian estimation methods are employed without the Kalman filter using a recursive algorithm to evaluate the likelihood function and are used to compare small-scale sticky-information and sticky-price DSGE models. Standard truncation methods are shown to not generally be innocuous.

1. Introduction

This paper presents a method for solving and estimating linear rational-expectations models with lagged expectations. Though the method itself contains little novelty, it contributes to the literature by combing several different methods established in the literature into one coherent approach. The resulting algorithm performs at least as well as each individual method while maintaining generality. The freely available software[1] strives to minimize computing and preprocessing time. I estimate simple sticky-information and sticky-price models using Bayesian methods, evaluating the likelihood function with an alternative to the Kalman filter. Two new contributions of this paper are the explicit consideration of models with infinite lagged expectations and the examination of truncation methods from the literature for such cases. The method and software should be of special interest to those interested in sticky information à la Mankiw and Reis (2002).

The solution method starts with the method of Taylor (1986), analyzing an infinite moving average solution. The undetermined coefficients approach yields a deterministic non-autonomous system of difference equations. After the largest expectational lag has been included, the system of difference equations becomes autonomous. Standard algorithms for solving potentially singular systems of difference equations are employed for the coefficients thereafter. Using the infinite moving average solution, the method will provide the unique, stable solution of the problem should it exist. The software provides the

☆ This research was originally entitled, "Solving linear rational-expectations models with lagged expectations quickly and easily".

* Tel.: +49 30 314 28912.
E-mail address: alexander.meyer-gohde@ww.tu-berlin.de

[1] MATLAB ® software and examples are available on the author's website.

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option of using the *QZ* method of Klein (2000) or the shuffle and eigenvalue method of Anderson and Moore (1985). The remaining coefficients are then determined by solving a block version of Mankiw and Reis's (2007) tridiagonal system. This particular synthesis eliminates the need for any manual reformulation and provides a computationally efficient algorithm that draws on preexisting algorithms with established properties.

For models with an infinite number of lagged expectations, e.g., models with the sticky-information Phillips curve of Mankiw and Reis (2002), the method of this paper uses an explicit convergence criterion to determine when and how to truncate. This is advantageous as current analyses of models containing infinite lagged expectations generally truncate either arbitrarily or through a process of trial and error. The appropriate truncation point will depend not only on the specific model, but also on the specific choice of parameter values. Using an arbitrary truncation point can provide potentially misleading results and using the truncation point derived from one particular parameter combination is unlikely to be suitable when parameter values are changed.

I estimate a simple New Keynesian model with Bayesian likelihood methods, comparing a sticky-information Phillips curve with its sticky-price equivalent. I treat the entire sample as a single draw from a multivariate normal distribution and obtain the covariance matrix using spectral methods. Evaluating the likelihood function requires the determinant and inverse of this matrix, which are calculated recursively using Akaike's (1973) Levinson method for block Toeplitz matrices. I am able to avoid the use of the Kalman filter, which is desirable due to the potentially prohibitive size of the underlying state space when many lagged expectations are present. A similar Levinson algorithm, familiar in the time series literature for the solution of the Yule–Walker equations and other aspects of ARMA estimation, (see Morettin, 1984, for a review) was also used by Leeper and Sims (1994, p. 99) to evaluate the likelihood function. The resulting estimates show that the sticky-information model can fare favorably in comparison with the standard sticky-price model, especially in reproducing the empirical lead of the output gap over inflation, and that arbitrary truncation can reverse this assessment.

Many solution methods for solving linear rational-expectations models can be found in the literature. For the analysis here, they can be split into two groups: those that explicitly allow for lagged expectations and those that do not. An incomplete list of the latter includes Blanchard and Kahn (1980), McCallum (1983), Anderson and Moore (1985), Binder and Pesaran (1995), King and Watson (1998), Uhlig (1999), Anderson (2010), Klein (2000), Sims (2001), and Christiano (2002). Although these methods can solve models with a finite number of lagged expectations, this requires the manual definition of dummy variables, see Binder and Pesaran (1995) or Sims (2001), to bring the system into canonical form.[2] The disadvantage is twofold. Firstly, defining dummy variables is tedious and prone to user error. Secondly, the computational burden from the increased number of variables can become prohibitive.

There are several solution methods that operate directly with lagged expectations. Taylor (1986) analyzes solutions that take the form of an infinite moving average and Mankiw and Reis (2002) demonstrate how this solution form can be applied to models with lagged expectations in the absence of forward-looking variables. Zadrozny (1998) provides a general method for solving systems with a finite number of lagged expectations, but the absence of a software implementation, as noted by Anderson (2008, p. 96), would require substantial work on behalf of the modeler to use his method. Wang and Wen's (2006) method solves models with lagged expectations by combining standard state-space techniques with a fixed-point approach for expectational errors. Requiring the modeler to manually reformulate lagged expectations as expectational errors reintroduces the potential for user error. Their fixed-point approach is unnecessarily complicated and computationally burdensome. Finally, Mankiw and Reis (2007) provide a method that works entirely on the infinite moving average representation. By reducing the system of equations to a scalar second-order non-autonomous difference equation and imposing a boundary condition at a finite horizon, they reduce the problem to solving a tridiagonal system. While the method could be altered to avoid the reformulation into a scalar system, it is unclear how and when the boundary conditions for a vector of variables should be imposed in more general settings. None of these methods give an explicit criterion for how to proceed when lagged expectations reach back into the infinite past.

The paper is organized as follows. Section 2 presents the model to be analyzed. Section 3 derives the solution method. Section 4 examines the dangers associated with truncations. Section 5 compares the method and its performance with alternate methods. Section 6 presents the method used for estimation and Section 7 examines the importance of lagged expectations using estimated sticky-information and sticky-price New Keynesian models. Finally, Section 8 concludes.

2. Statement of the problem

Log-linearized economic models can typically be represented by a system of linear expectational difference equations:

$$0 = \sum_{i=0}^{I} A_i E_{t-i}[Y_{t+1}] + \sum_{i=0}^{I} B_i E_{t-i}[Y_t] + \sum_{i=0}^{I} C_i E_{t-i}[Y_{t-1}] + \sum_{i=0}^{I} F_i E_{t-i}[W_{t+1}] + \sum_{i=0}^{I} G_i E_{t-i}[W_t] \tag{1}$$

$$W_t = \sum_{j=0}^{\infty} N_j \varepsilon_{t-j}, \quad \varepsilon_t \sim i.i.d.\ \mathcal{N}(0, \Omega) \tag{2}$$

[2] Christiano (2002, p. 23) does allow for the information set to vary across equations, but to have varying information sets within one equation, dummy definitions would still be necessary.