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Indeterminacy and the elasticity of substitution in one-sector models

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ABSTRACT

This paper introduces a new production externality via factor substitution and explores its effects on generating indeterminacy in one-sector growth models. With the elasticity of substitution depends on the average level of capital intensity, indeterminacy is possible as long as the steady-state level of capital is below the normalized level of the CES production function. Given that the elasticity of factor substitution is decreasing in capital and the marginal product of capital is decreasing in terms of the elasticity, indeterminacy can occur because efficient factor substitution from capital deepening offsets the diminishing returns of capital.

1. Introduction

It has been well known that under certain market imperfection conditions such as external effects, dynamic macroeconomic models can be subject to indeterminacy; that is, with the same initial condition there exists a continuum of equilibria all converging to a common steady state.[1] Boldrin and Rustichini (1994) provide a complete characterization on the indeterminacy conditions for dynamic one-sector models. It is shown that these conditions for indeterminacy of one-sector models are very hard to satisfy under standard specification of preferences and technologies.[2] In their seminal work, by allowing for endogenous labor supply in a one-sector growth model with Cobb–Douglas technology, Benhabib and Farmer (1994) show that indeterminacy can occur but with large positive factor externalities. In particular, labor externalities have to be so large that the social returns to labor should be greater than unity and the labor demand curve should be upward-sloping.

These empirically implausible externalities, as admitted by Benhabib and Farmer (1999), lead us to reconsider how factor externalities should be introduced into the production technology.[3] Recent efforts are put to consider more general specifications on technology and preferences in order to lower the requirement on return to scale and factor externalities.

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[1] See Benhabib and Farmer (1999) for an extensive survey of the literature on indeterminacy and sunspots.

[2] It is necessary to have both negative factor externality and increasing social marginal product for the possibility of indeterminacy. With standard Cobb–Douglas production technology, these necessary conditions cannot be met. Kehoe (1991) generates a numerical example of indeterminacy using very special quadratic production technology and utility function.

[3] Alternative ways of generating indeterminacy in dynamic models are considered by others in the literature. One popular way is to extend the analysis to a multi-sector setting; for example, see Mino (2001), Nishimura and Venditti (2004) and Chen and Lee (2007).

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Table 1
Review on EOS estimates (US).

Study	Sample	Assumption on technological change	Estimated EOS
Arrow et al. (1961)	1909–1949	Hicks-neutral	0.57
Brown and DeCani (1963)	1890–1918	Factor augmenting	0.35
	1919–1937		0.08
	1938–1958		0.11
	1890–1958		0.44
David and van de Kundert (1965)	1899-1960	Factor augmenting	0.32
Bodkin and Klein (1967)	1909–1949	Hicks-neutral	0.5–0.7
Wilkinson (1968)	1899–1953	Factor augmenting	0.5
Sato (1970)	1909–1966	Factor augmenting	0.5–0.7
Panik (1976)	1929–1966	Hicks-neutral	0.76
Berndt (1976)	1929–1968	Hicks-neutral	0.96–1.25
Kalt (1978)	1929–1967	Factor augmenting	0.76
Antras (2003)	1948–1998	Hicks-neutral	0.94–1.02
		Factor augmenting	0.8

Source: All the studies given in the table can be found in Klump et al. (2004).

However, most of the studies on indeterminacy that allow for non-unity elasticities of substitution between capital and labor seems to require the elasticity to be much larger than the Cobb–Douglas case. For example, by adopting a general constant elasticity of substitution (CES) technology, Pintus (2006) finds that with large enough the elasticity of capital–labor substitution (in the range of [2.16, 13.37]), the elasticity of intertemporal substitution in consumption (in the range of [6.67, 25]) as well as the elasticity of labor supply (infinity; indivisible labor), local indeterminacy can arise under small labor externalities.[4] However, it was 40 years ago when Arrow et al. (1961) introduced the CES production function and pointed out that the elasticity was significantly less than one. Afterwards, this magnitude of the elasticity has been confirmed by numerous cross-sectional and aggregate time-series empirical studies.[5] As summarized by Acemoglu (2009):

> Let us first note that in the context of capital-labor substitution, the empirical evidence suggests that an elasticity of substitution of $\sigma<1$ is much more plausible…An elasticity less than 1 is not only consistent with the available empirical evidence, but it is also economically plausible. An elasticity of substitution between capital and labor greater than 1 would imply that production is possible without labor or without capital, which appears counterintuitive. (Introduction to Modern Economic Growth, p. 519)

Borrowed from Klump et al. (2004), Table 1 summarizes the estimated elasticities of substitution based on US data in the literature. The majority view is that the elasticity of substitution is less than unity (with the largest estimate being 1.25). In face of the empirical studies, we can conclude that requiring either a high returns to scale or a high elasticity of substitution makes the possibility of indeterminacy not promising. In this paper, under constant returns to scale in production, local indeterminacy can occur when the elasticity of substitution between capital and labor is within the empirically plausible range. We believe that our findings provide important value added to the existing literature.

Another important feature of our paper is that it takes a different route to investigate indeterminacy in one-sector growth models. Specifically, we focus on a novel mechanism where externalities affect the substitution between factor inputs. The idea that the elasticity of substitution varies with capital deepening is not new. For instance, in his classic book of *The Theory of Wages*, Hicks (1963) explains the possibility of a falling elasticity of substitution when capital accumulates. Hicks writes:

> . . . we examine . . . two extreme cases. In both we shall assume population constant and capital increasing; but in one technical progress is very lethargic, in the other very rapid.
> In the first case, where inventions of all kinds are almost wholly absent, substitution is practically confined to …the increased use of those commodities requiring much capital, and the more extensive use of known capitalistic methods. It is conceivable that in an early stage these may be sufficient to keep the elasticity of substitution greater

[4] The consensus is to have the social returns to scale to be below the upper bound of 1.09 estimated by Basu and Fernald (1997). For extensions to non-separable preferences, see Pintus (2007) and Lloyd-Braga et al. (2006). Another interesting feature of the latter is that the formulation is characterized by constant returns at both the private and social levels, as in the current paper. We also note that Drugeon (2008) has studied the same constant-return property in a two-sector model with factor externalities.

[5] For instance, see Lovell (1973) for a classic study on manufacturing sectors; for recent updates, see Ramcharran (2001), Claro (2003) and Kouliavtsev et al. (2007). We also note that in some mutli-input studies, the estimated elasticity of substitution between capital and non-capital input can be larger than one. See Griffin and Gregory (1976) and Krusell et al. (2000). But the estimated elasticity of substitution between capital and labor (e.g., skilled labor in Krusell et al. (2000)) is found to be less than one. Finally, see Acemoglu (2009) for a survey of the elasticity estimate and Klump et al. (2004) for a discussion on the possible bias in time series studies.