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Excess covariance and dynamic instability in a multi-asset model

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ABSTRACT

The presence of *excess covariance* in financial price returns is an accepted empirical fact: the price dynamics of financial assets tend to be more correlated than their fundamentals would justify. We advance an explanation of this fact based on an intertemporal equilibrium multi-assets model of financial markets with an explicit and endogenous price dynamics. The market is driven by an exogenous stochastic process of dividend yields paid by the assets that we identify as market *fundamentals*. The model is rather flexible and allows for the coexistence of different trading strategies. The evolution of assets price and traders' wealth is described by a high-dimensional stochastic dynamical system. We identify the equilibria of the model consistent with a baseline assumption of procedural rationality. We show that these equilibria are characterized by excess covariance in prices with respect to the dividend process. Moreover, we show that in equilibrium there is a positive expected marginal profit in choosing more risky portfolios. As a consequence, the evolutionary pressure generates a trend towards more remunerative strategies, which, in turn, increase the variance of prices and the dynamic instability of the system.

1. Introduction

A huge econometric literature, started in the first part of the '80, suggests that price returns in financial markets are characterized by volatility levels too high to be explained by corresponding movements in the fundamental value of assets or by macroeconomic variability in terms of aggregate consumption or money supply (LeRoy, 2008). The high "unexplained" volatility of prices of financial securities has been taken by many as suggestive of a intrinsic instability in the functioning of speculative markets. Along similar lines, several recent studies investigate whether the observed high degree of covariance in price returns can be explained by a similar degree of correlation among the economic values of the traded assets (Bouchaud and Potters, 2003). As discussed in Marsili et al. (2011), a remarkable finding is that this covariance exhibits different behaviors at different timescales. In practice, however, the phenomenon of *excess covariance* is difficult to measure, because of the vagueness of what can be identified as fundamentals in the real world: what is the original covariance above which we find an additional excess covariance? There are attempts in the literature to identify proxies of these fundamentals. Shiller (1989), Froot and Dabora (1999) and Brealey et al. (2010), each for a different historical period, have shown that the same asset traded in different markets tend to behave differently, following in each location the trend of the local market. The appropriate way of disentangling this empirical fact from arbitrage

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opportunities and from differences due to the spread of information is still an open question. Kallberg and Pasquariello (2008) propose a purely empirical approach. They analyze the assets' prices of 82 firms, from different sectors, traded on the NYSE: filtering the 1976–2001 data from aggregate shocks they obtain idiosyncratic data that, they show, are much less correlated than the global financial ones. Regardless of any possible objection that could be made towards the choice of the *fundamentals*, all these papers confirm the evidence of an excess of covariance in the financial markets, compared to what is normally observed in the real economy.

Several models have been proposed to explain the appearance of excess covariance. Most of them focus on the behavior of agents: traders tend to be correlated in their activity and consequently induce an analogous correlation in assets' prices. Kyle and Xiong (2001) propose a model in which the different wealth effects of traders induce some homogeneity in portfolio choices. Kodres and Pritsker (2002) assume that the correlation is instead due to the re-balancing activity of risk averse agents. Yuan (2005) assumes that it is the effect of financial constraints, while Veldkamp (2006) imagines that information is costly and introduces a herding behavior. Finally, Marsili and Raffaelli (2006) describe a stochastic behavioral model in which a single type of agents adjusts dynamically a single portfolio with a mean–variance strategy.

In this paper we show that it is possible to obtain the same result in a very general setting as a direct effect of trading, independently on any collinearity in the strategies of different traders. We do it by formalizing a stylized model of financial markets with a multiplicity of coexisting trading strategies, and a multiplicity of traded assets. We assume an exogenous underlying *real* economy, that we simply identify with a stream of random dividends paid by every asset. The dividend process is governed by a multivariate stationary distribution which is described by a constant variance–covariance matrix. This matrix represents the covariance structure of the fundamental value of the traded assets. The model assumes a Walrasian endogenous asset pricing for all risky assets through market clearing and an inelastic supply of a riskless security which acts as the numeraire of the economy.

The model we propose fits in the rich literature of Heterogeneous Agents Model (HAM) (see Hommes, 2006 for a fairly recent survey) but is much less restrictive in terms of agents trading behavior and does not constraint the analysis to a subset of boundedly rational trading strategies. There are only a few attempts that deal with multiple-assets framework within the HAM literature (see for instance Westerhoff, 2004; Chiarella et al., 2005, 2010) but none of them, to the best of our knowledge, analyze the emergence of excess covariance.

The dynamics of our model is described by a high-dimensional stochastic dynamical system. In the HAM literature these systems are typically analyzed by "switching the noise off", i.e., by replacing the driving stochastic process with its expected values and considering the corresponding deterministic skeleton (see for instance the analysis in Anufriev et al., 2006 and Anufriev and Bottazzi, 2010). The argument supporting the use of the deterministic skeleton for the analysis of the stochastic dynamics runs typically as follows. If the deterministic skeleton converges to an asymptotically stable fixed point, the stochastic processes are also "close" to the corresponding fixed values, provided that the noise is reasonably small (e.g., when the dividend yields are i.i.d. and their support belong to the basins of attraction of the fixed point). On the other hand, if the deterministic skeleton exhibits a bifurcation in which the fixed point loses its stability, the associated random system is analogously perturbed away from the fixed point. In a model with multiple assets, however, the previous deterministic skeleton approach has the important drawback of not allowing for the analysis of higher moments of the fluctuation around equilibria. Indeed the correlations between the price returns of different assets are always zero in the deterministic approximation. Clearly, an alternative approach is needed. One possible solution is to directly characterize the stochastic or deterministic fixed points of the complete random dynamical system and study their stability using stochastic version of the Hartman–Grobman theorem. This is the approach followed by the evolutionary finance literature, recently surveyed in Blume and Easley (2009) and Evstigneev et al. (2009). These methods become however very complicated if one wants to adopt more realistic dividend processes and abandon the assumption of a finite number of states of the world. Indeed we are not aware of any study of markets with long-lived multiple assets, continuous support of the dividend structure, and investment strategies with a feedback from the past returns. Bottazzi and Dindo (2010) partially fill the gap introducing direct price feedback in agents' strategy, but only discuss the case of short-lived assets. In this paper we try to follow an intermediate approach between the full-fledged analysis of the stochastic system, which is unfeasible, and the study of the deterministic skeleton, which is too crude. We exploit the notion of Procedurally Consistent Equilibria (PCE) already introduced in Anufriev et al. (2006) Anufriev and Bottazzi (2010) and Anufriev and Dindo (2010) for single-asset markets. In PCE the unique requirement is that in equilibrium the investment shares of agents are constant. The model is then closed by the rationality assumption that realized prices are consistent with the assumption on which the investment shares were chosen.

The structure of the paper is as follows. In Section 2 we present the model, introducing the exogenous dividend process, the market clearing mechanism and discussing the range of admissible strategies. We look at price dynamics and growth wealth in the more general setting, without specializing to any particular behavioral rule. In Section 3 we study the PCE of the model and we derive a system describing the co-evolution of the agent's wealth and the assets' returns. We show that the returns' correlation matrix can be decomposed into two terms: the first is the correlation matrix of the dividend process, while the second accounts for the excess correlation. This excess correlation is endogenously created and depends, in general, on the agents' investment strategies. Analogously, we show that there exists endogenous excess return for different assets, which implies that the equity premium can simply be the result of endogenous investment process. In Section 3.3 we study the evolutionary stability of PCE equilibria with respect to an invasion by a different strategy. We show that, under the most general conditions, in this