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Testing conditional asymmetry: A residual-based approach ☆

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ABSTRACT

We propose three residual-based tests for conditional asymmetry. The distribution is assumed to fall into the class of skewed distributions of Fernández and Steel (1998). In this class, asymmetry is measured by the ratio between the probabilities of being larger and smaller than the mode. Estimation is performed under the null hypothesis of constant asymmetry of the innovations and, in a second step, tests for conditional asymmetry are performed on generalized residuals through parametric and non-parametric methods. We derive the asymptotic distribution of the tests that incorporates the uncertainty of the estimated parameters. A Monte Carlo study shows that neglecting this uncertainty severely biases the tests. An empirical application on a basket of daily returns reveals that financial data often present dynamics in the conditional skewness.

1. Introduction

We propose parametric and nonparametric residual-based tests for conditional asymmetry. We assume that observations follow a dynamic location–scale model with error term following a law in the class of skewed distributions of Fernández and Steel (1998). Estimation is done under the null hypothesis of constant skewness and the tests are performed on the residuals. These tests are, in spirit, similar to the Breusch–Godfrey and Engle's ARCH tests for the mean and the variance. There are, however, two important differences. First, the tests presented here are not moment-based. While both the Breusch–Godfrey test and Engle's ARCH test are based on (auto)regressions for the first and second conditional moments, we do not test for (auto)correlation on the third moment. Rather, we test for (auto)correlation in the asymmetry, defined as the ratio between the probabilities of being larger or smaller than the mode. Second, we take into account the uncertainty of the estimated parameters. As the tests are residual-based, their distributions depend on the

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estimated parameters. We rely on Pierce (1982) to derive the correct asymptotic distributions of the tests. Neglecting the uncertainty coming from the first step estimators severely distorts the size of the test, and, hence, the reliability of the corresponding decisions.

From a financial perspective, this paper fits within the research that incorporates asymmetry and kurtosis into, among others, asset pricing and portfolio allocation. Harvey and Siddique (2000) propose a modified mean–variance asset pricing model where conditional skewness is priced. The assumption that investors prefer right skewed portfolios to left skewed portfolios justifies the inclusion of the third moment. In their empirical application, they find conditional left skewness to be economically significant, increasing the risk premia by 3.6%. Chabi-Yo et al. (2006) study the importance of conditional skewness when estimating a skewness premium. Based on Samuelson (1970), they study the optimal asset allocation in a mean–variance–skewness framework. They also introduce a separation theorem based on a fund of the market portfolio, which is a function of the skewness, and analyze the risk compensation in the presence of conditional asymmetry. Dittmar (2002) and Guidolin and Timmermann (2006) expand the standard CAPM to a four-moment CAPM, including skewness and kurtosis. The former shows that its explanatory power for cross-section US stock returns raises considerably. The latter explains the home country bias in investments, concluding that, while the standard mean–variance approach assigns 30% of stocks to domestic markets, adding both skewness and kurtosis raises this percentage to 70%.

From an econometric perspective, this paper fits into a branch of the literature that proposes tests and models for conditional symmetry. Zheng (1998) presents a nonparametric kernel-based test for conditional symmetry. Bai and Ng (2001) propose a distribution free conditional symmetry test that is valid for nonstationary and non-*i.i.d.* observations. In a similar vein, Delgado and Escanciano (2007) propose an omnibus test for conditional symmetry in dynamic models. The main feature of this test is that it allows for unknown functional forms of higher conditional moments. Hong and Li (2005) and Egorov et al. (2006) present an omnibus nonparametric evaluation test for conditional density models that explicitly takes into account the impact of parameter estimation uncertainty. As for the models, several papers (Coroneo and Veradas, forthcoming; Ghysels et al., 2011) propose conditional quantile-based asymmetry as an extension of the Bowley coefficient. Hansen (1994) estimates conditional asymmetry and kurtosis by extending the ARMA and GARCH model to the third and fourth order moments. Jondeau and Rockinger (2003) extend the generalized Student-t distribution of Hansen (1994) and investigate the presence of conditional skewness and kurtosis, which are modeled as a function of lagged innovations. In their application to FX markets, they detect the presence and persistence of conditional asymmetry, while the probability mass on the tails is relatively constant.

As the last two references, our approach is also parametric. We assume that observations follow a dynamic location–scale model with error term following a skewed distribution on the class of Fernández and Steel (1998). Since our focus are financial applications, in the Monte Carlo study and in the empirical application we illustrate our main results on the skewed-t distribution, a skewed version of the Student-t. This law has a tractable density function, which facilitates maximum likelihood, and four parameters that explain the main features of a probability law for financial returns: location, scale, asymmetry and tail thickness.[3] The skewing mechanism is based on a scale parameter that is introduced inversely on each side of the mode. Asymmetry is therefore defined in terms of the ratio of probabilities of being to the left and right of the mode.

This skewing parameter is our main object of interest. We estimate the model under the null hypothesis of constant asymmetry, i.e. time invariant skewing parameter. Under this hypothesis, the sequence of ratios of the conditional probabilities of being to the left and right of the mode should be independent. Following Engle and Manganelli (2004), these ratios are equivalent to a sequence of indicator functions that take value one if the t-th observation is lower than the mode and zero otherwise. We test for independence of this sequence of binary variables both parametrically and nonparametrically. We first consider Wald-type tests based on generalized linear models. Detailed derivations are provided for two important cases: the linear probability and the logistic regression models. Second, we consider a nonparametric Wald–Wolfowitz runs test. In both cases, we take into account the uncertainty from the estimation of the parameters of the skewed distribution. Using the results from Pierce (1982), we derive the correct asymptotic distribution of the tests under the null.[4]

Pierce (1982) is not the only possible way to account for parameter uncertainty. Engle and Manganelli (2004) propose the in-sample dynamic quantile test, which takes into account the parameter uncertainty from the conditional quantile model. Hong and Lee (2003) also propose a diagnostic test for linear and nonlinear time series models, where the parameter uncertainty has no impact on the limit distribution of the test statistic. Accuracy of out-of-sample predictions can also be affected by parameter uncertainty. Olmo (2008), MacCracken (2000) and West (1996) explore the problem and propose test statistics with a distribution that incorporates the parameter uncertainty.

In a comprehensive Monte Carlo study, we evaluate the properties of the tests for different specifications of the conditional mean and variance, and for different sample sizes. We show that the tests are correctly sized provided that the uncertainty in the parameter estimation is taken into account. We also show that the power of the test is acceptable for a reasonable sample size. As a robustness check, we show the properties of the tests under

[3] Hansen (1994) proposes a similar distribution, though the derivation is different.
[4] Tse (2002) also uses Pierce's correction in a residual-based test for conditional heteroskedasticity.