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# Global structural optimization considering expected consequences of failure and using ANN surrogates

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ABSTRACT

The literature is filled with structural optimization articles which claim to minimize costs but which disregard the costs of failure. Due to uncertainties, minimum cost can only be achieved by considering expected consequences of failure. This article discusses challenges in solving real structural optimization problems, taking into account expected consequences of failure. The solution developed herein combines non-linear FE analysis (by positional FEM), structural reliability analysis, Artificial Neural Networks (used as surrogates for objective function) and a hybrid Particle Swarm Optimization algorithm, which efficiently solves for the global optimum. Optimization of a steel-frame transmission line tower is the application example.



## 1. Introduction

Optimization of real structural engineering systems is a demanding task. Modeling the structural behavior of real structures requires numerical models (e.g., FEM) of many degrees of freedom, which are computationally expensive to evaluate. Optimizing such structures requires hundreds to thousands of structural response evaluations. And the resulting optimal structures must be robust with respect to the uncertainties inherently present in loads and in the strength of structural materials.

In a competitive environment, structural systems have to be designed taking into account not just their functionality, but also expected construction and operational costs, and their capacity to generate profits. This capacity can be adversely affected by the costs of failure. Expected costs of failure quantitatively represent the different risks that construction and operation of a given facility imply to the owner, to users, to employees, to the general public and/or to the environment. Uncertainty implies risk, and the possibility of undesirable structural responses.

In monetary terms, risk (or the expected cost of failure) is given by the product of failure probabilities by failure costs. Failure probabilities and failure consequences can be directly affected by structural design.

In structural engineering, economy and safety are generally considered to be competing goals. To the conventional structural engineer, increasing safety implies greater costs, and reducing costs may compromise safety. Hence, designing structural systems would involve a tradeoff between safety and economy. In common engineering practice, this tradeoff is addressed subjectively. When using structural design codes, the tradeoff has already been decided by a code committee, which specifies safety coefficients to be used in design, and basic safety measures to be adopted in construction and operation. In deterministic structural optimization, this tradeoff is completely neglected, because failure probabilities are not quantified. In classical [1–16] Reliability-Based Design Optimization (RBDO) the tradeoff between safety and economy is also not addressed, because failure probabilities are constraints and not part of the objective function. Robust design optimization [17,18] searches for designs which are less sensitive to the existing uncertainties, but safety-economy tradeoffs are also not addressed.

When expected costs of failure are included in the design equation [19–27], one realizes that economy and safety are, in fact, not competing goals. Safety is just another design variable which directly affects expected costs of failure. Since failure probabilities and consequences of failure are directly affected by structural design [28], optimum (minimum cost) design can only be achieved by quantifying uncertainties, probabilities of failure and costs of failure. In other words, optimum (minimum cost) design can only be achieved by quantifying expected costs of failure, and by treating safety as (an indirect) design variable [28]. This is called structural risk optimization herein and in a few other Refs. [27–29].

Optimization of real structural engineering systems is a demanding task. Even more demanding is the optimization of real structural engineering systems in consideration of the several

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sources of uncertainty which may affect system performance. Quantifying failure probabilities due to these uncertainties involves structural reliability analyses, which require repetitive solutions of the "deterministic" numerical FE models. When Monte Carlo simulation is used, these may reach thousands to millions for a single reliability analysis. Special subset simulation schemes have been devised [6] for solving optimization problems under uncertainty, but these are only effective for reduced numbers of design variables. When the efficient First Order Reliability Method (FORM) method is used for reliability analysis, still hundreds to thousands structural responses may be needed for each reliability analysis. When FORM is used, a nested optimization problem is obtained [9–16]. For classical RBDO problems [1–16], where failure probabilities are constraints and not design variables, a number of approaches have been proposed to avoid the nested optimization loops [9–16]. However, these shortcuts do not apply to risk optimization problems.

The authors are not aware of any similar shortcuts to solving structural risk optimization problems. Hence, each step in a risk optimization solution requires at least one complete reliability analysis, which represents hundreds to thousands of structural response evaluations.

Moreover, it was found that risk optimization problems possess many local minima [28]. Hence, local optimization algorithms can at best improve a given initial design. Finding the (global) optimum demands global optimization algorithms. This significantly increases the difficulties, as global optimization algorithms require evaluations of the objective function throughout the design space, which is more demanding than local optimization. Each objective function evaluation in risk optimization leads to a complete reliability analysis, which requires many structural (mechanical) response evaluations. Hence, the increase in computational cost is compounding. In this article, a special hybrid BFGS–PSO algorithm [30] is used to solve the global optimization problem. As direct solution of this problem is very expensive, Artificial Neural Networks (ANN) are used as surrogate models for the objective function, in order to reduce the computational burden.

Another compounding difficulty arises when using global optimization algorithms to solve structural optimization problems. Local algorithms search for a better solution in a vicinity of a given initial design, which is normally a feasible and well-behaved design. Global optimization algorithms, on the other hand, have to test designs that are scattered all over the design space. Hence, it is easy to arrive at weird structural configurations, which would make no sense to a structural designer, but which end up being tested by the optimization algorithm. Weird designs can even belong to the failure domain, that is, they are more likely to fail than not. These weird designs can lead to numerical instabilities and convergence difficulties for both the non-linear (FE) mechanical models and for the reliability analysis algorithms.

In the present article, the positional finite element method is used to compute non-linear structural responses [31–36]. The positional FE method is a robust numerical analysis method, as it allows computing large displacements under material non-linearities. In the positional FEM, the displaced configuration is the primary unknown: displacements and rotations are evaluated afterwards. Equilibrium equations are evaluated in the displaced configuration. Material points are located by configuration-change functions and their gradients. Non-linear Cauchy-Green deformation measures are used, as well as their energy conjugate. Second-order Piola–Kirchhoff stress tensors are considered. With these stress–strain measures, the Saint Venant–Kirchhoff constitutive relation is obtained. Stationarity of the total potential energy is used to arrive at the equilibrium equations. The Newton–Raphson method is used to solve the non-linear equations, with a consistent tangent stiffness matrix.

The article is laid out as follows. In Section 2, the risk optimization problem is formulated. Section 3 describes two different Artificial Neural Networks, which are used as surrogates to aid solution of global risk optimization problems, as described in Section 4. Section 5 presents results for an application problem, involving the optimization of a powerline tower subject to random wind loads. Concluding remarks are presented in Section 6.

The practical problem addressed in this article, and the main difficulties to overcome, have strong similitude with some of the viewpoints defended by Prof. G.I. Schuëller, to whom this *Special Issue* is dedicated. Prof. G.I. Schuëller has been a strong advocate that reliability analysis should be made compliant with large FE models [37–43] and high stochastic dimensions [44–52]. Moreover, Prof. Schuëller has been co-author of a number of significant articles on structural optimization under uncertainties [53,49,54–57] and has authored important benchmark reviews on this topic [16,17]. The real structural engineering example addressed herein does not qualify as a truly large FE model, nor does it include high stochastic dimensions. However, the example pushes on three other important dimensions of the optimization problem: many design variables, consideration of expected costs of failure and solution for the global optimum.

## 2. Formulation: risk optimization problem

### 2.1. Structural reliability problem

Let $\mathbf{X}$ and $\mathbf{d}$ be vectors of structural system parameters. Vector $\mathbf{X}$ represents all random or uncertain system parameters, and includes geometric characteristics, resistance properties of materials or structural members, and loads. Some of these parameters are random in nature; others cannot be quantified deterministically due to uncertainty. Typically, resistance parameters can be represented as random variables and loads are modeled as random processes of time. Vector $\mathbf{d}$ contains all deterministic design variables, like nominal member dimensions, partial safety factors, design life, parameters of inspection and maintenance programs, etc. Vector $\mathbf{d}$ may also include some parameters of random variables in $\mathbf{X}$; for instance, the mean of a random variable may be a design variable.

The existence of uncertainty implies risk, that is, the possibility of undesirable structural responses. The boundary between desirable and undesirable structural responses is given by limit state functions $g(\mathbf{X}, \mathbf{d}) = 0$, such that:

$$\Omega_f(\mathbf{d}) = \{\mathbf{x}|g(\mathbf{x},\mathbf{d}) \leqslant 0\} \quad \text{is the failure domain}$$

$$\Omega_s(\mathbf{d}) = \{\mathbf{x}|g(\mathbf{x},\mathbf{d}) > 0\} \quad \text{is the survival domain} \tag{1}$$

Each limit state describes one possible failure mode of the structure, either in terms of serviceability or ultimate capacity. The probability of undesirable structural response, or probability of failure, is given by:

$$P_f(\mathbf{d}) = P[\mathbf{X} \in \Omega_f(\mathbf{d})] = \int_{\Omega f} (\mathbf{d}) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \tag{2}$$

where $f_{\mathbf{x}}(\mathbf{x})$ is the joint probability density function of vector $\mathbf{X}$. The probabilities of failure for individual limit states and for system failure can be evaluated using traditional structural reliability methods such as FORM, SORM and Monte Carlo simulation [58,59].

In the risk optimization problem, reliability analyses have to be repeated thousands of times. Hence, the algorithm used for reliability analysis has to be very efficient. In this paper, reliability analyses are performed by the First Order Reliability Method (FORM), which is reasonably accurate and quite efficient. Importantly, the efficiency of FORM is equivalent for structural configurations leading