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On the use of a class of interior point algorithms in stochastic structural optimization

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In this paper the feasibility of using a particular feasible direction interior point algorithm for solving reliability-based optimization problems of high dimensional stochastic dynamical systems is investigated. The optimal design problem is formulated in terms of an inequality constrained non-linear optimization problem. A class of interior point algorithms based on the solution of the first-order optimality conditions is considered here. For this purpose, a quasi-Newton iteration is used to solve the corresponding nonlinear system of equations. Several numerical examples are presented to illustrate the feasibility of the proposed methodology.

1. Introduction

Structural optimization by means of deterministic mathematical programming techniques has been widely accepted as a viable tool for engineering design [1,2]. However in many structural engineering applications response predictions are based on models whose parameters are uncertain. Despite of the fact that traditional approaches have been used successfully in many practical applications, a proper design procedure must explicitly consider the effects of uncertainties as they may cause significant changes in the global performance of final designs [3–6]. Under uncertain conditions probabilistic approaches such as reliability-based formulations provide a realistic and rational framework for structural optimization which explicitly accounts for the uncertainties [7–16]. It is noted that, however, alternative approaches do exist as well. For example methodologies based on non-traditional uncertainty models can be very useful in a number of cases [17–21].

In the present contribution a reliability-based formulation is considered. In particular structural design problems involving dynamical systems under stochastic loadings are analyzed. The optimization problem is formulated as the minimization of an objective function subject to multiple design requirements including standard deterministic constraints and reliability constraints. First excursion probabilities are used as measures of system reliability. The corresponding reliability problems are expressed as multidimensional probability integrals involving a large number of uncertain parameters. Such parameters describe the uncertainties in the structural properties and excitation.

In the field of reliability-based optimization of stochastic dynamical systems several procedures have been recently developed allowing the solution of quite demanding problems [22–29]. The numerical efforts associated with the solution of this type of problems is dominated by the reliability assessment step. Therefore one strategy is to construct approximate representations of the quantities depending on the uncertain parameters as an explicit function of the design variables [5,24,30,31]. On the other hand, direct stochastic search algorithms have also proved to be useful tools for solving challenging stochastic reliability-based optimization problems [26,32–34]. While the use of the above optimization approaches has been found useful in a number of structural optimization problems there is still room for further developments in this area. It is the objective of this contribution to evaluate the feasibility of using a class of interior point algorithms in the context of reliability-based optimization problems of high dimensional stochastic dynamical systems. In particular, an optimization scheme based on the solution of the first-order optimality conditions is considered here [35–38]. Based on this optimization scheme different applications are presented to illustrate the potentially of the design process in realistic engineering problems.

The organization of the paper is as follows. Section 2 presents the formulation of the reliability-based optimization problem to be studied in this contribution. The basic ideas of the optimization algorithm used in the present formulation are discussed in Section 3. Section 4 addresses several implementation and numerical aspects of the proposed optimization scheme. One test problem and two application problems are presented in Section 5. The paper closes with some final remarks.

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2. Description of the problem

2.1. Optimization problem

Consider the following inequality constrained non-linear optimization problem

$$\begin{aligned} &\mathrm{Min}_{\mathbf{x}} f(\mathbf{x}) \\ &\text{s.t. } g_i(\mathbf{x}) \leqslant 0, \quad i = 1, \ldots, n_c \\ &\qquad s_i(\mathbf{x}) \leqslant 0, \quad i = 1, \ldots, n_r \\ &\qquad \mathbf{x} \in X \end{aligned} \tag{1}$$

where $\mathbf{x}$, x_i, $i = 1,\ldots,n_d$ is the vector of design variables with side constraints $x_i^l \leqslant x_i \leqslant x_i^u$, $f(\mathbf{x})$ is the objective function, $g_i(\mathbf{x}) \leqslant 0$, $i = 1,\ldots,n_c$ are the standard constraints, and $s_i(\mathbf{x}) \leqslant 0$, $i = 1,\ldots,n_r$ are the reliability constraints. It is assumed that the objective and constraint functions are smooth functions of the design variables. The objective function f can be defined in terms of initial, construction, repair or downtime costs, structural weight, structural performances, etc. The standard constraints are related to general design requirements such as geometric conditions, material cost components, availability of materials, etc. Finally, the reliability constraints are associated with design specifications characterized by means of reliability measures. Reliability measures given in terms of failure probabilities with respect to specific failure criteria such as serviceability and partial or total collapse failure are considered in the present formulation. Throughout this formulation it is assumed, for simplicity, that the evaluation of the objective function $f(\mathbf{x})$ and the standard constraint functions $g_i(\mathbf{x})$, $i = 1,\ldots,n_c$ is numerically inexpensive, while the evaluation of the reliability constraints is considerable more involved. However, objective and standard constraint functions that are numerically expensive to evaluate can also be considered in the present formulation. Therefore, the above formulation is quite general in the sense that different stochastic optimization formulations can be considered.

2.2. Reliability measures

For structural systems under stochastic excitation the probability that design conditions are satisfied within a particular reference period T provides a useful reliability measure. Such measure is referred as the first excursion probability and quantifies the plausibility of the occurrence of unacceptable behavior (failure) of the structural system. For example, for upper bound constraints of the responses the failure event F can be defined as $F(\mathbf{x},\mathbf{z}) = D(\mathbf{x},\mathbf{z}/h) > 1$, where D is the so-called normalized demand function defined as

$$D(\mathbf{x}, \mathbf{z}/h) = \max_{j=1,\ldots,l} \max_{t\in[0,T]} \frac{h_j(t, \mathbf{x}, \mathbf{z})}{h_j^*} \tag{2}$$

where $\mathbf{z} \in \Omega_{\mathbf{z}} \subset R^{n_z}$ is the vector of uncertain variables involved in the problem, $h_j(t,\mathbf{x},\mathbf{z})$, $j = 1,\ldots,l$ are the response functions associated with the failure event F, and $h_j^* > 0$ is the acceptable response level for the response h_j. In this context the quotient $h_j(t,\mathbf{x},\mathbf{z})/h_j^*$ can be interpreted as a demand to capacity ratio, as it compares the value of the response $h_j(t,\mathbf{x},\mathbf{z})$ with the maximum allowable value of the response h_j^*. A similar characterization of the normalized demand function can be obtained if the constraints of the responses are given in terms of lower bounds. In this setting, it is clear that the responses h_j are functions of time (due to the dynamic nature of the excitation), the design variable vector $\mathbf{x}$, and the random vector $\mathbf{z}$. These functions are obtained from the solution of the equation of motion that characterizes the structural model. The uncertain system parameters $\mathbf{z}$ are modeled using a prescribed probability density function $p(\mathbf{z})$. This function indicates the relative plausibility of the possible values of the uncertain parameters $\mathbf{z} \in \Omega_{\mathbf{z}}$. It is noted that the vector of uncertain variables describes all uncertainties involved in the problem, that is, model and loading parameters.

2.3. Reliability constraints

With the previous notation the reliability constraint functions are written in terms of failure probability functions as

$$s_i(\mathbf{x}) = P_{F_i}(\mathbf{x}) - P_{F_i}^*, \quad i = 1, \ldots, n_r \tag{3}$$

where $P_{F_i}(\mathbf{x})$ is the probability function for the failure event F_i evaluated at the design $\mathbf{x}$, and $P_{F_i}^*$ is the target failure probability for the i^{th} failure event. The failure event F_i is characterized in terms of the demand function D_i as before, where $D_i(\mathbf{x},\mathbf{z}) = D(\mathbf{x},\mathbf{z}/h^i)$, and $h_j^i(t, \mathbf{x}, \mathbf{z})$, $j = 1, \ldots, l_i$ are the response functions associated with the failure event F_i. The failure probability function $P_{F_i}(\mathbf{x})$ evaluated at the design $\mathbf{x}$ can be written in terms of the multidimensional probability integral

$$P_{F_i}(\mathbf{x}) = \int_{D_i(\mathbf{x},\mathbf{z})>1} p(\mathbf{z}) d\mathbf{x} \tag{4}$$

It is noted that the multidimensional probability integral involves a large number of uncertain parameters (hundreds or thousands) in the context of dynamical systems under stochastic excitation [39–42]. Therefore, the reliability estimation for a given design constitutes a high-dimensional problem which is extremely demanding from a numerical point of view [43–47].

3. Optimization strategy

3.1. General description

A first-order optimization scheme based on feasible directions is selected in the present implementation. In particular, a class of feasible direction algorithms based on the solution of the Karush–Kuhn–Tucker (KKT) first-order optimality conditions is considered here [36,37]. At each iteration the search direction is a descent feasible direction of the objective function. A one-dimensional line search is then carried out in order to obtain a new feasible design better than the previous one. The process continues until convergence is achieved. By construction the method generates a sequence of steadily improved feasible designs. This class of algorithms has proved to be quite effective in deterministic optimization problems [48,49]. In fact, a large number of test problems [50] have been solved very efficiently without any change in the code and with the same set of parameters that characterizes the algorithm. In addition validation calculations have shown that the number of iterations remains comparable when the size of the problem is increased.

3.2. Basic ideas

3.2.1. Optimality conditions

The KKT first-order optimality conditions corresponding to the inequality constrained optimization problem (1) can be expressed as [37]

$$\begin{aligned} &\nabla f(\mathbf{x}) + \nabla \mathbf{g}(\mathbf{x})\boldsymbol{\lambda}_g + \nabla \mathbf{s}(\mathbf{x})\boldsymbol{\lambda}_s = \mathbf{0} \\ &\mathbf{G}(\mathbf{x})\boldsymbol{\lambda}_g = \mathbf{0}, \quad \mathbf{S}(\mathbf{x})\boldsymbol{\lambda}_s = \mathbf{0} \\ &g_i(\mathbf{x}) \leqslant 0, \quad i = 1, \ldots, n_c, \quad s_i(\mathbf{x}) \leqslant 0, \quad i = 1, \ldots, n_r \\ &\boldsymbol{\lambda}_g \geqslant \mathbf{0}, \quad \boldsymbol{\lambda}_s \geqslant \mathbf{0} \end{aligned} \tag{5}$$

where $\boldsymbol{\lambda}_g \in R^{n_c}$ and $\boldsymbol{\lambda}_s \in R^{n_r}$ are the vectors of dual variables, $\nabla \mathbf{g}(\mathbf{x}) \in R^{n_d \times n_c}$ and $\nabla \mathbf{s}(\mathbf{x}) \in R^{n_d \times n_r}$ are the matrices of derivatives of