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Dynamic instability in generic model of multi-assets markets ☆

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ABSTRACT

We introduce a generic model of a multi-asset financial market, which takes into account the impact of portfolio investment on price dynamics. This captures the fact that financial correlation determine the optimal portfolio but are affected by investment based on it. We show that, under very general conditions, such a feedback on correlations gives rise to an instability when the volume of investment exceeds a critical value. Close to the critical point the model exhibits dynamical correlations very similar to those observed in empirical data.

1. Introduction

Financial markets allow individuals to cope with risk (Elton and Gruber, 1995; Markowitz, 1959). The main intuition is that *diversification* of investment across different stocks reduces risk. The efficiency of diversification as well as the investment strategies themselves, however, depends on the degree of correlation between the returns of different assets. Indeed stock returns exhibit a relatively high degree of comovement, as summarized by the capital asset pricing model, whose simplest practical implementation—the one factor model—postulates that stocks' covariance arises from the effect of the same factor: the market portfolio (Elton and Gruber, 1995). Still stock prices comove far in excess than their common fundamentals (Pindyck and Rotemberg, 1993). This effect has been related to several effects: for example the correlation of a stock with other stocks of an index increases when the former is also included in the index (Barberis et al., 2004). Indeed, the dynamics of prices is way more complex than that of correlated random walks, as correlations themselves exhibit a non-trivial dynamics. This has led to various generalizations of stochastic volatility (GARCH) models, such as the Dynamic Conditional Correlation model (Engle, 2002), which explain much better empirical data than Constant Conditional Correlation models (see Bauwens et al., 2006, for a review).

The same issue has been addressed in a different strand of literature, with tools and techniques borrowed from physics (Bouchaud and Potters, 2003). The strong tendency of stocks' prices to fluctuate with "the market" arises, in this context, as

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a strong separation of the largest eigenvalue of the correlation matrix from all the others (Potters et al., 2005). The largest eigenvalue of the correlation matrix, in turn, undergoes a non-trivial dynamics with sharp rises during crashes (Drozdz et al., 2001). Actually the structure (Bonanno et al., 2003) and dynamics (Potters et al., 2005) of financial correlations is much more complex than that captured by a single eigenvalue and postulated by one factor models (see also Onnela et al., 2003; Borghesi et al., 2007).

This paper focuses, in particular, on the time dependence of financial correlations. Its working hypothesis is that non-stationary correlations arise as a consequence of the impact of trading in portfolio strategies. Under this hypothesis, optimal portfolio strategies, which depend on financial correlations, also induce correlations in the underlying stocks because they generate correlated demand across different stocks. Our aim is to capture, within a stylized model, the feedback loop induced by portfolio strategies through correlations of financial returns.

This idea combines elements which have been recently discussed in the economics literature. On one side, the fact that excess comovement of stocks arises as a consequence of correlated demand has received some empirical support (Greenwood, 2005). On the other, non-stationarity in financial correlations has been related to exogenous illiquidity shocks in the case of few assets (Brunetti and Caldarera, 2006), or to the impact of few big investors on the price dynamics (Bank and Baum, 2004). By contrast, our approach suggests that non-stationary correlations might have a systemic origin related to the impact of a large number of traders, all following similar optimal portfolio strategies, in order to diversify investment across a large number of stocks.

A simple realization of this idea has been considered in Raffaelli and Marsili (2006), which has shown the existence of a systemic instability when the impact of trading in optimal portfolio strategies is large enough. Interestingly, close to the instability the model generates realistic time series, with fat tailed returns and non-stationary correlations. Such systemic fragility implies a extreme susceptibility to external shocks suggesting that the market response may be largely amplified by the instability. In this paper, we extend this approach to a broad class of models. We find that the existence of a systemic instability due to the impact of optimal portfolio strategies is quite robust. It holds for any combination of portfolios, independent of their underlying risk measure or of the use of noise filtering techniques, i.e. is not related to the instability of popular risk measures discussed in Kondor et al. (2007). The origin of the instability is identified in the limit where agents estimate risk and return from an infinitely long time series. In this limit averages converge to expectations, as under the law of large numbers, only in a region of the parameter space, which we shall call the stable phase. Out of this region, i.e. in the unstable phase, non-linearities give rise to extreme fluctuations for which the law of large numbers does not hold.

We characterize the boundary of the stable phase, which allows us to discuss in detail what makes markets more or less stable in our model (see Section 4.1). For example, one remarkable result is that markets where agents are more risk averse tend to be closer to the instability, i.e. riskier. The dynamics of the model can be studied in a systematic expansion, with respect to the inverse of the typical time-scale over which agents estimate historical data. This makes it possible to characterize the singular behavior close to the instability. More precisely, we are able to compute the critical exponents governing the singular behavior of different quantities as well as to unravel the mechanism which leads to wild fluctuations at the critical point.

The paper is organized as follows. First we set the stage by presenting, in a case study on historical data from NYSE, the range of phenomena which we shall be interested in. The model is introduced in Section 3, where we also discuss its behavior via numerical simulations, and studied in Section 4. The final section presents some concluding remarks.

2. Background: notations and empirical results

Let $\mathbf{x}_t \equiv (x_{1,t}, x_{2,t}, \ldots, x_{N,t})'$ denote the (column) vector of log-prices, at time t, of the N assets composing the market, i.e. $x_{i,t} = \log p_{i,t}$ (here and in the following the transpose operation is denoted with a prime $'$). Time is discrete and typically we shall consider the time-scales of a day or longer. Let us also introduce time averages for a generic time dependent quantity a_t

$$E_t^{(\mu)}[a] = \mu \sum_{k=0}^{\infty} (1-\mu)^{t-k} a_{t-k} = (1-\mu) E_{t-1}^{(\mu)}[a] + \mu a_t. \tag{1}$$

Here the constant $\mu \in (0,1)$ modulates the range $\tau = 1/|\log(1-\mu)|$ over which the average extends. In particular, the limit $\mu \to 0$ corresponds to averages over an infinite time range. When needed, a superscript (μ) will be used in what follows to stress the dependence on the time-scale of averaged quantities.

So, for example, the average return can be defined as

$$\mathbf{r}_t^{(\mu)} = E_t^{(\mu)}[\mathbf{x}_t - \mathbf{x}_{t-1}], \tag{2}$$

and the empirical covariance matrix reads

$$\hat{C}_t^{(\mu)} = E_t^{(\mu)}[\delta\mathbf{x}_t \times \delta\mathbf{x}_t'], \tag{3}$$

where $\delta\mathbf{x}_t = \mathbf{x}_t - \mathbf{x}_{t-1} - \mathbf{r}_t^{(\mu)}$ is the vector of excess returns, and we introduce the direct product $\times$ between two vectors, defined so that the element i, j of $\mathbf{a} \times \mathbf{b}'$ is $a_i b_j$.