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Estimation of evolutionary spectra for simulation of non-stationary and non-Gaussian stochastic processes

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ABSTRACT

No spectral representation-based methodology exists to simulate non-stationary and non-Gaussian stochastic processes. This is due to the inability to determine a unique evolutionary spectrum (ES) for a process with known non-stationary autocorrelation. Here, a framework is developed to estimate evolutionary spectra for non-Gaussian processes so that realizations may be simulated using spectral representation. Two cases are considered. First, the non-Gaussian ES is estimated for a process with prescribed Gaussian ES and marginal non-Gaussian probability density function (PDF). In the second case, a compatible underlying Gaussian ES is estimated for a process with incompatible prescribed non-Gaussian ES and marginal PDF.

1. Introduction

Monte Carlo Simulation is the most accurate and robust technique available for solving certain classes of problems which are stochastic in nature and involve strong nonlinearities, instabilities, and/or very large magnitude of the uncertain system parameters. Monte Carlo Simulation has become much more feasible for solving increasingly complex problems in recent times due to dramatic improvements in computational power. Many of these problems require the generation of sample realizations of stochastic processes and fields as input. With this motivation, a number of techniques have been developed to rapidly and accurately simulate stochastic processes and fields of various types.

However, sample function generation of many processes and fields is not trivial and, in general, the ease of simulation depends largely on the nature of the process/field. For instance, several well established methodologies which are conceptually straightforward and computationally efficient exist for simulation of stochastic processes/fields which are both stationary/homogeneous and Gaussian. Simulation of non-stationary/non-homogeneous and Gaussian processes/fields is more difficult but possible using methods such as the Spectral Representation Method (SRM) [1,2] and Karhunen–Loéve (K–L) expansion [3] among others. Furthermore, a number of techniques have been developed to simulate stationary/homogeneous and non-Gaussian processes and fields such as those utilizing translation process/field theory (e.g. [4–12]), spectral correction (e.g. [13–15]), and Polynomial Chaos/Hermite Polynomial transformations (e.g. [16–18]) among others. The aforementioned techniques vary in their accuracy, conceptual simplicity (or complexity), and computational efficiency. A description of these techniques is beyond the scope of this paper.

To date though, few models exist for simulation of stochastic processes/fields which are both non-stationary/non-homogeneous and non-Gaussian-and no such spectral representation-based model is available. Those models which do exist utilize the polynomial chaos representation and/or Karhunen–Loeve expansion such as the models proposed by Sakamoto and Ghanem [19,20] and Phoon et al. [21,22]. The main objective here is to model a general non-stationary/non-homogeneous and non-Gaussian process/field as a translation process/field and generating sample functions using the SRM. As already mentioned, several methods have been developed to simulate a general stationary/homogeneous non-Gaussian process/field as a translation process/field (see [4–12]). However, the extension of these models to simulating non-stationary/non-homogeneous processes/fields is not trivial. In fact, there are significant theoretical barriers to developing such a technique (even when the prescribed evolutionary power spectral density (or evolutionary spectrum – ES) and the non-Gaussian marginal probability density function (PDF) are compatible according to the non-stationary extension of translation process/field theory established by Ferrante et al. [23]). This paper addresses these barriers and outlines a new methodology to approximate a general

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non-stationary/non-homogeneous and non-Gaussian process/field by a non-stationary/non-homogeneous translation process/field.

The paper begins with a review of the theoretical framework necessary to develop the proposed technique. This starts with a brief review of the SRM with specific focus on the simulation of non-stationary/non-homogeneous and Gaussian processes/fields. Next, translation process/field theory is reviewed with an emphasis on its applications to non-stationary/non-homogeneous processes/fields. Within this context, the theoretical challenges to merging these theories and producing a comprehensive model for non-stationary/non-homogeneous and non-Gaussian processes/fields are outlined. Then, a new approximate technique is introduced which allows the translation, with reasonable accuracy in most cases, between Gaussian and non-Gaussian evolutionary spectra. Two cases are considered. The first case (the so-called "forward" problem) involves estimating the non-Gaussian ES from a known underlying Gaussian ES. The second and practically more interesting case (herein referred to as the "inverse" problem) involves estimating the underlying Gaussian ES from an arbitrarily prescribed incompatible (according to translation process/field theory) pair of non-Gaussian ES and marginal probability density function (PDF). Once the underlying Gaussian ES has been estimated, the SRM may be used to generate sample realizations of the process/field.

As a note to avoid ambiguity, stochastic processes and fields will hereafter be referred to only as processes, although all ideas and concepts developed are equally applicable to fields.

2. Spectral Representation Method for simulation of Gaussian and non-stationary stochastic processes

The Spectral Representation Method (SRM) for simulation of scalar Gaussian and non-stationary processes was first proposed by Shinozuka and Jan [1]. The SRM relies on the theory of evolutionary power developed by Priestley [24]. Therefore, prior to introducing the simulation formula itself, a brief review of this theory is provided.

2.1. Evolutionary power spectrum

A general one-dimensional, zero-mean, uni-variate stochastic process $X(t)$ may be expressed as [24]:

$$X(t) = \int_{-\infty}^{\infty} \phi(\omega, t) dZ(\omega) \tag{1}$$

where $Z(\omega)$ is an orthogonal process with:

$$E[|dZ(\omega)|^2] = d\mu(\omega) \tag{2}$$

if there exists a family of functions $\{\phi(\omega,t)\}$ and a measure $\mu(\omega)$ such that the autocovariance function $Cov(s,t)$ (equivalently autocorrelation function ACF, $R(s,t)$) admits the representation [24]:

$$Cov(s,t) = R(s,t) = E[X(s)X(t)] = \int_{-\infty}^{\infty} \phi(\omega, s)\phi^*(\omega, t) d\mu(\omega) \tag{3}$$

where the asterisk denotes the complex conjugate. In general, a multitude of different families of functions exist such that the process may be represented as in Eq. (1). The selection of a family of functions to represent the process is analogous to the selection of a basis for a vector space. Therefore, this representation is not unique. For stationary processes, one convenient family of functions is the complex exponentials:

$$\phi(\omega, t) = e^{I\omega t} \tag{4}$$

where I is the imaginary unit and ω is the circular frequency, which provides the well-known spectral representation.

However, the complex exponential family is not valid for representation of non-stationary processes. One family of functions which may be used to represent non-stationary processes that preserves the physically useful concept of frequency is the family of amplitude modulated complex exponentials given by Priestley [24]:

$$\phi(\omega, t) = A(\omega, t) e^{I\theta(\omega)t} \tag{5}$$

The family of functions $\phi(\omega,t)$ in Eq. (5) is said to be oscillatory for some $\theta(\omega)$ if the modulating function can be written as [24]:

$$A(\omega, t) = \int_{-\infty}^{\infty} e^{It\theta} dH(\omega, \theta) \tag{6}$$

with $|dH(\omega,\theta)|$ having absolute maximum at $\theta = 0$. If $\theta(\omega)$ is a single-valued function of ω, then the oscillatory process $X(t)$ may be represented as:

$$X(t) = \int_{-\infty}^{\infty} A(\omega, t) e^{I\omega t} dZ(\omega) \tag{7}$$

with ACF given by:

$$R(s,t) = \int_{-\infty}^{\infty} A(\omega, s) A^*(\omega, t) e^{I\omega\tau} d\mu(\omega) \tag{8}$$

where $\tau = s - t$ is referred to as the lag or separation between time instants s and t.

Given an oscillatory process with the representation in Eq. (7), the evolutionary power spectrum (or evolutionary spectrum-ES) is defined as [24]:

$$dS(\omega, t) = |A(\omega, t)|^2 d\mu(\omega) \tag{9}$$

Priestley notes that it is convenient to "standardize" the modulating function $A(\omega,t)$ such that $A(\omega,0) = 1$. Consequently, the measure $\mu(\omega)$ represents the "initial spectrum" (at $t = 0$) and $|A(\omega,t)|^2$ represents the change in time of the spectrum relative to the initial spectrum.

It will be assumed here that $A(\omega,t)$ is a real function and that $\mu(\omega) = 1$ such that the evolutionary spectrum can be expressed as:

$$S(\omega, t) = A^2(\omega, t) \tag{10}$$

with ACF:

$$R(s,t) = \int_{-\infty}^{\infty} \sqrt{S(\omega, s) S(\omega, t)} e^{I\omega\tau} d\omega. \tag{11}$$

The basic advantage of the evolutionary spectrum is that it can be interpreted in the same way as the classical power spectrum – i.e. power distribution over frequency-for the oscillatory process at time t assuming that the process is "semi-stationary." The process is called semi-stationary [24] if $A(\omega,t)$ is a slowly-varying function of t. Specifically, $A(\omega,t)$ is considered slowly-varying if its Fourier Transform shown in Eq. (6) is "highly concentrated" in the region of zero frequency. The width of $dH(\omega,\theta)$ is measured by the function [24]:

$$B_{\mathcal{F}}(\omega) = \int_{-\infty}^{\infty} |\theta| |dH(\omega, \theta)| \tag{12}$$

and the process is considered semi-stationary if $B_{\mathcal{F}}(\omega)$ is bounded for all ω. The semi-stationary process has a (functional family specific) characteristic width $B_{\mathcal{F}}$ defined by Priestley [24]:

$$B_{\mathcal{F}} = \frac{1}{\sup_{\omega}(B_{\mathcal{F}}(\omega))} \tag{13}$$

The absolute characteristic width of the process considers all families of functions admitting the representation in Eq. (7) and may be expressed as: