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The Euro/Dollar exchange rate: Chaotic or non-chaotic? A continuous time model with heterogeneous beliefs ☆

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ABSTRACT

The aim of this paper is to develop a continuous time exchange rate model that allows for heterogeneity of the agents' beliefs, in order to explore non-linearities and possible chaotic behaviour. The theoretical model contains an intrinsic non-linearity that gives rise to a jerk differential equation, which is in principle capable of generating chaos. The model is econometrically estimated in continuous time with Euro/Dollar data and examined for the possible presence of chaotic motion. Our results indicate that the possibility of chaotic dynamics in our model is rejected.

1. Introduction

The aim of this paper is to investigate the possibility of chaotic dynamics in the Euro/Dollar exchange rate in the presence of heterogeneous beliefs. The interest of economists in chaos theory started in the 1980s, more than 20 years after the onset of this theory in physics, which is conventionally dated from 1963 when the meteorologist E.N. Lorenz published his paper on what became to be known as the Lorenz (1963) attractor.

A few words are in order to explain the interest of economists in chaos and the exchange rate. After the failure of the standard structural models of exchange rate determination in out-of-sample ex-post forecasts (the most notable empirical rejection is that by Meese and Rogoff, 1983a,b; for subsequent studies see Gandolfo et al., 1990, 1993; Gandolfo, 2002; Rogoff and Stavrakeva, 2008), the exchange rate has come to be considered as a stochastic phenomenon, and exchange rate forecasting has come to rely on technical analysis and time series procedures, with no place for economic theory. Economic theory can be reintroduced in various ways, one of which is through a chaotic model. In fact, this would explain the

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apparently erratic behaviour of the exchange rate not through purely stochastic processes, but as due to a deterministic economic model capable of generating chaos. Another possibility would be to use a non-linear non-chaotic but stochastic structural model.

Furthermore, it has become evident that it is not possible to understand exchange rate behaviour by relying on models with representative agents. All forms of this simplifying approach have failed empirically (see Sarno and Taylor, 2002). There is now abundant evidence that market participants have quite heterogeneous beliefs on future exchange rates. These different expectations introduce non-linear features in the dynamics of the exchange rate. Heterogeneous agent models may create complex endogenous dynamics, including chaotic dynamics. This approach was initiated by Frankel and Froot (1987, 1990a,b). Further studies developed this line of research mainly in the context of stock markets (e.g., Kirman, 1991; Day and Huang, 1990; Brock and Hommes, 1997, 1998; Lux, 1998; Le Baron et al., 1999; Gaunersdorfer et al., 2008); for surveys of this field of literature, see Hommes (2006), Chiarella et al. (2009), Hommes and Wagener (2009), Lux (2009), and Westerhoff (2009).

The empirical evidence in favour of chaos in the exchange rate is not very strong. Sometimes chaos has been detected in the data (see Bajo-Rubio et al., 1992; De Grauwe et al., 1993; Chen, 1999; Bask, 2002; Brzozowska-Rup and Orlowski, 2004; Weston, 2007; Torkamani et al., 2007) but most often no such dynamics has been found (Brooks, 1998; Guillaume, 2000; Federici and Gandolfo, 2002; Serletis and Shahmoradi, 2004; Resende and Zeidan, 2008). In general, the empirical evidence for chaotic dynamics in economic time series is very fragile.

Studies aimed at detecting chaos in economic variables can be roughly classified into two categories.

(I) On the one hand, there are studies that simply examine the data and apply various tests, such as the studies mentioned above (for applications to the exchange rate see Bajo-Rubio et al., 1992; Cuaresma, 1998; Guillaume, 2000, Chapter 3; Schwartz and Yousefi, 2003; Vandrovych, 2006; Weston, 2007). These tests have been originally developed in the physics literature. This approach is not very satisfactory from our point of view. In fact, as pointed out by a referee, even if the presence of chaos were detected, we would not know whether the detected phenomenon was produced by the economy, or perhaps from outside (for example, it is well known to climatologists that the weather can be chaotic, and the weather certainly has some influence on the economy). Consistently with this point of view, our approach aims at finding the dynamic chaotic model (if any) suggested by economic theory that underlies the data.

Besides, in the case of the investigation of individual time series to determine whether they are the result of chaotic or stochastic behaviour, the single blind comparative study of Barnett et al. (1997) showed that the tests compared are inconclusive about chaos coming from within the economy.

(II) On the other hand, structural models are built and analysed. This analysis can in principle be carried out in several ways:

(II.a) Showing that plausible economic assumptions give rise to theoretical models having dynamic structures that fall into one of the mathematical forms known to give rise to chaotic motion;
(II.b) Building a theoretical model and then:
(II.b_1) Giving plausible values to the parameters, simulating the model, and testing the resulting data series for chaos; or
(II.b_2) Estimating the parameters econometrically, and then proceeding as in b_1.

Existing chaotic exchange rate models (De Grauwe and Vansanten, 1990; Reszat, 1992; De Grauwe and Dewachter, 1993a,b; De Grauwe et al., 1993; De Grauwe and Grimaldi, 2006a,b; Ellis, 1994; Szpiro, 1994; Chen, 1999; Da Silva, 2000, 2001; Moosa, 2000, Chapter 9) follow approaches (II.a) or (II.b_1). From the theoretical point of view, these models show that with orthodox assumptions (PPP, interest parity, etc.) and introducing non-linearities in the dynamic equations, it is possible to obtain a dynamic system capable of giving rise to chaotic motion. A reason for implementing approach (II.b_2) as well, is that, when simulating the model for different values of the parameters, it is possible to use the confidence intervals around the point estimates as a guide (see Section 5).

In the present paper, after a preliminary investigation of the data according to (I),[1] we follow approach (II.a) and approach (II.b_2). The exchange rate can be taken as a continuous-time phenomenon, hence we build a model whose dynamics is based on differential equations. This model allows for heterogeneity of the agents' beliefs and possesses an *intrinsic* non-linearity, which is in principle capable of generating a chaotic motion.

After the analysis of its theoretical properties, the model is econometrically estimated in continuous time with Euro/Dollar data and examined for the possible presence of chaotic motion.

2. The model: formulation in terms of excess demands for foreign exchange

Our starting point is that the exchange rate is determined in the foreign exchange market through the demand for and supply of foreign exchange. This is a truism, but it should be complemented by the observation that, when all the sources

[1] In order to detect the presence of chaos, in the first step, we use tick-by-tick Euro/Dollar exchange rate from January 2003 to December 2009 (1-min and 5-min intervals). Similarly to many other papers, we study the exchange rate returns (the exchange rate return at time *t* is calculated as the log difference of two consecutive exchange rate levels). Tools from dynamical systems theory, such as the maximum Lyapunov exponent, are used. In addition we apply the reshuffled (surrogate) data procedure, which is unfortunately overlooked in most tests carried out in economic studies. The results of this analysis indicate that the data do not possess the features that are required to classify them as chaotic.