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Isoparametric closure elements in boundary element method

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ABSTRACT

An innovative method is proposed for constructing isoparametric boundary elements to simulate closed surfaces. These elements are named "isoparametric closure elements" and can not only accurately simulate spherical, elliptical, and other closed surface geometries, but also interpolate physical quantities defined over these surfaces. As a result of using the proposed closure elements, each of these surfaces can be discretized into only one element along the circumferential direction. A number of closure elements having 4–26 nodes are investigated to examine the computational error, and three are recommended to be used in the boundary element method (BEM) analysis. These closure elements are applied to BEM analysis of heat conduction and solid mechanics problems. A technique for eliminating singularities involved in boundary integrals over closure elements is also presented. A number of numerical examples will be given to demonstrate the computational accuracy and efficiency of the proposed closure elements.

1. Introduction

Mechanical behaviors of composite materials with inclusions have received a considerable amount of attention [1–6]. Inclusions may induce rapidly changing stresses in composite structures and across boundaries of different material types, and thus may result in serious consequences in composite and inhomogeneous structures. In order to assess the mechanical behavior of composite materials with inclusions, numerical methods, such as the finite element method (FEM) [7,8] and boundary element method (BEM) [4,9], are often used to compute the displacements and stresses across the interfaces of inclusions and the surrounding media.

In the FEM analysis of inclusion problems, the size of the discretized element should be on the same order as the diameter of the inclusions. Therefore, many volume elements are required to accurately simulate the inclusions. This drawback has restricted the capability of FEM to solve composite structures with many inlaid inclusions. Due to its lower dimensions of discretization and high computational precision, BEM is another important numerical analysis tool which has been widely used in solving many engineering problems [10,11]. Since problems with inclusions are multiple media problems, Multi-Domain BEM (MDBEM) [12–14] is usually used to solve these problems. In MDBEM, the boundary integral equations are applied to each medium and the final system of equations is formed by assembling the algebraic equations of all media involved. Recently, some researchers have been investigating the use of a single integral equation to solve multi-media problems [9,15], which may have the advantage of being easier coded than with MDBEM. When using all the above-mentioned methods to solve problems with inclusions, the boundaries of the inclusions are discretized using the quadratic boundary element. To fit the highly curved geometry of the inclusions, a significant number of elements are required in discretization. This inevitably results in a huge system of equations, especially in solving modern structures which tend to contain many inclusions. To reduce the number of boundary elements needed to discretize closed boundaries, some BEM researchers have turned their attention to the development of hole-like boundary elements [16–22]. Henry and Banerjee in 1991 [16] constructed a 3 node hole element to approximate variations in displacement along the circumference of a hole. Following this idea, Buroni and Marczak [21] derived higher order shape functions of hole elements with 4–6 nodes. These shape functions are expressed in terms of trigonometric functions and were derived from a truncated Fourier series in which the series coefficients are obtained by solving the linear system in which unitary values are imposed to each node and null values to the remaining nodes. In order to evaluate boundary integrals over these hole elements using the Gaussian quadrature, the variable (angle ϑ) in trigonometric functions needs to be expressed

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in terms of the intrinsic coordinate (ξ) through using a variable substitution technique [22].

Recent years, isogeometric methods have become attractive tools in numerical simulation. Beer et al. [23–25] gave a detailed description of the NURBS based boundary element method to treat geotechnics problems and made comparisons to isoparametric elements. The attractiveness of these methods stems from the fact that the description of the geometry can be taken directly from CAD programs, avoiding the need for mesh generation. The weakness of these methods is that the geometry of the problem is defined by some control points which are different from the collocation points served to define physical quantities.

In this paper, a new family of high-order isoparametric hole and closure elements is constructed based on the Lagrange polynomial interpolation formulation and the closure conditions at the two ends of an arc. These elements can be directly expressed using the intrinsic coordinates and can easily be used in BEM code. Three numerical examples are presented to verify the correctness and robustness of the constructed high-order isoparametric elements. The key feature of the derived closure elements is that only one element is needed to simulate the geometry and interpolate physical quantities over an inclusion. Therefore, the number of elements and nodes needed can be greatly reduced compared to existing quadratic boundary elements. The novelties of the paper can be summarized as follows:

- The idea of constructing shape functions for a closed line based on the Lagrange polynomial interpolation formulation is presented for the first time, which provides a basis of forming high order isoparametric hole elements.
- Shape functions listed in the paper for 6, 10, 14, 20, and 26 node isoparametric closure elements are new, which have not yet been seen in the literature.
- A new element sub-division method is proposed for eliminating singularities involved in the boundary integrals over the proposed closure elements. This method is performed in the intrinsic parameter space and suitable for any types of boundary elements.

2. Boundary integral equations for elasticity and heat conduction problems

In three-dimensional elasticity problems, the boundary integral equation can be expressed as [10,11]

$$cu_i(P) = \int_\Gamma U_{ij}(P,Q)t_j(Q)d\Gamma(Q) - \int_\Gamma T_{ij}(P,Q)u_j(Q)d\Gamma(Q) \tag{1}$$

where $c = 1/2$ for smooth boundary points and $c = 1$ for interior points; P and Q represent the source and field points, respectively u_j and t_j are displacements and tractions defined on the boundary Γ; U_{ij} and T_{ij} are Kelvin's fundamental solutions for displacements and tractions, which can be expressed as

$$U_{ij} = \frac{1}{16\pi(1-\nu)\mu r}[(3-4\nu)\delta_{ij} + r_{,i}r_{,j}] \tag{2a}$$

$$T_{ij} = \frac{-1}{8\pi(1-\nu)r^2}\left\{\frac{\partial r}{\partial n}[(1-2\nu)\delta_{ij} + 3r_{,i}r_{,j}] + (1-2\nu)(n_i r_{,j} - n_j r_{,i})\right\} \tag{2b}$$

where μ is the shear modulus, ν is the Poisson ratio, and r is the distance between the source point and field point.

To evaluate the boundary integrals included in Eq. (1), the boundary Γ of the problem is discretized into a series of boundary elements [10]. The coordinate, traction and displacement over each element can be expressed in terms of their nodal values through shape functions:

$$\begin{aligned} x_j &= \sum_{k=1}^{m} N_k x_j^k \\ t_j &= \sum_{k=1}^{m} N_k t_j^k \\ u_j &= \sum_{k=1}^{m} N_k u_j^k \end{aligned} \tag{3}$$

where m is the number of element nodes, N_k is the shape function for the k-th node, and x_j^k, t_j^k and u_j^k are the values of coordinate, traction and displacement of the j-th component at node k. Fig. 1 shows two types of frequently used linear and quadratic boundary elements over the boundary of a three-dimensional problem.

Assuming that the boundary Γ of the problem is discretized into n_e boundary elements and using Eq. (3), the discretized form of Eq. (1) can be rewritten as

$$\begin{aligned} cu_i(P) = &\sum_{e=1}^{n_e}\left\{\sum_{k=1}^{m} t_j^k \int_{\Gamma_e} U_{ij}(Q,P)N_k(Q)d\Gamma(Q)\right\} \\ &- \sum_{e=1}^{n_e}\left\{\sum_{k=1}^{m} u_j^k \int_{\Gamma_e} T_{ij}(Q,P)N_k(Q)d\Gamma(Q)\right\} \end{aligned} \tag{4}$$

Integrating Eq. (4) over each element, an algebraic equation can be obtained. When the source point P is collocated at all boundary nodes and applying boundary conditions to the specified traction and displacement nodes, a system of equations about boundary unknowns can be set up. In solving the system, the unknown tractions and displacements can be obtained.

For heat conduction problems, similar integral equations to Eqs. (1) and (4) can also be set up. Interested researchers may go to related references [10,26].

If the boundary elements shown in Fig. 1 are used to simulate a hole boundary, many elements are required to fit the geometry. To overcome this drawback, a family of special isoparametric boundary elements are constructed in the following section. Only one of these elements is needed to simulate an entire hole, and, therefore, considerable computational nodes can be reduced.

3. Construction of isoparametric closure boundary elements

In this section, a family of isoparametric closure elements is constructed based on the Lagrange polynomial interpolation formulation, which can be used to interpolate physical and geometrical quantities over the surface of an inclusion. Firstly, the isoparametric hole element is constructed, and then a tube element is formed by extruding the hole element along the longitudinal direction. Finally, the closure element is constructed by merging some nodes of the tube element.

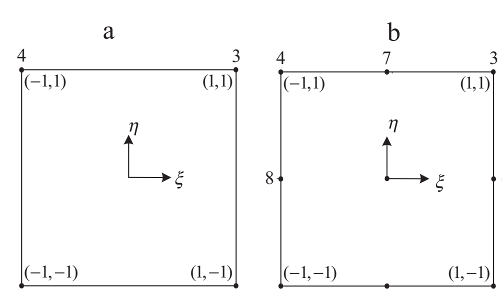

Fig. 1. Frequently used surface elements: (a) 4 node linear and (b) 8 node quadratic.