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Cournot duopoly when the competitors operate multiple production plants

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ABSTRACT

This article considers a Cournot duopoly under an isoelastic demand function and cost functions with built-in capacity limits. The special feature is that each firm is assumed to operate multiple plants, which can be run alone or in combination. Each firm has two plants with different capacity limits, so each has three cost options, the third being to run both plants, dividing the load according to the principle of equal marginal costs. As a consequence, the marginal cost functions come in three disjoint pieces, so the reaction functions, derived on basis of global profit maximization, may also consist of disjoint pieces. This is reflected in a particular bifurcation structure, due to border-collision bifurcations and to particular basin boundaries, related to the discontinuities. It is shown that stable cycles may coexist, and the *non-existence of unstable cycles* constitutes a new property. We also compare the coexistent short periodic solutions in terms of the resulting real profits.

1. Introduction

The present paper contributes to the analysis of Cournot oligopoly models where each of the competitors has the option of utilizing several plants. Economists have been interested in oligopoly models for many years. After the dynamic adjustment process proposed by Cournot (1838), several contributions followed to show that the Cournot equilibrium may be unstable (see e.g. Palander, 1939; Theocharis, 1960; Fisher, 1961; McManus and Quandt, 1961) or not unique (Robinson, 1933; Palander, 1936, 1939). The literature dealing with Cournot models has increased greatly in the last two decades. The existence and stability of Cournot equilibria have been analyzed by Furth (1986), via a cubic marginal costs curve, Bonanno (1988), via a cubic marginal revenues curve, and local/global stability properties have become important issues.

A rich literature stream on this subject is related to duopoly or triopoly models, usually described by continuous and smooth reaction functions (see e.g. Okuguchi, 1976; Al-Nowaihi and Levine, 1985; Dana and Montrucchio, 1986; Puu, 1991, 1996; Kopel, 1996). A peculiarity of these models is the existence of *multistability*. This means that several attracting sets coexist, and the long run behavior of the game may give rise to different alternatives (see e.g. Agiza, 1999; Agiza et al., 1999; Bischi and Naimzada, 1999; Bischi et al., 2000; Bischi and Kopel, 2001; Puu and Sushko, 2002). In these models the local bifurcations (changes in the stability of the equilibria or the cycles) are quite standard and well known, being related to smooth changes in the eigenvalues of the linear approximation of the dynamic model. Moreover, global properties (i.e. not

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related to the eigenvalues) are used to investigate the shape and qualitative changes of the basins of attraction. In these models the basins are bounded by the stable sets of saddle points or unstable equilibria.

Besides these *smooth models*, some authors proposed duopoly games characterized by piecewise smooth or even discontinuous reaction functions. In these cases, the dynamic analysis may involve the occurrence of the so-called *border-collision bifurcations*, which are related to the crossing of the equilibria or cycle points through sets where the dynamical system is not differentiable, that separate regions where the maps that represent the dynamical system are differentiable but different. These bifurcations may cause sudden stability switches and/or the appearance/disappearance of cycles. The study of border-collision bifurcation is not new in models applied to economics (see e.g. the pioneering works by Hommes, 1991, 1995; Hommes and Nusse, 1991; Hommes et al., 1995). The applications of such techniques have been used in business cycle models and in oligopoly models for a few years (see e.g. Gallegati et al., 2003; Puu et al., 2002, 2005; Sushko et al., 2003; Puu and Sushko, 2006). It is worth mentioning that the main results on this subject are provided by Nusse and Yorke (1992, 1995), Maistrenko et al. (1993, 1995, 1998), Di Bernardo et al. (1999), Banerjee et al. (2000a, b), Halse et al. (2003), and Zhanybai et al. (2003), with applications to economics also, as in Sushko et al. (2005, 2006).

Cournot duopoly models with unstable equilibria and discontinuities were already introduced by Furth (1986) and even before by Palander (1939), who also proposed several situations where multistability could occur. An example is a case with a piecewise linear Robinson-type demand function where the elasticity increased drastically when price was lowered. Then the marginal revenue function becomes *discontinuous with jumps*, and the corresponding reaction function is discontinuous as well. Moreover, the reaction functions could intersect in several points, Cournot (or Nash) equilibria, giving rise to multistability. This case was reconsidered in detail by two of the present authors (see Puu et al., 2002).

Palander also developed a case where each firm could operate several production plants, some suitable for small scale production (low fixed costs, high marginal costs), others suitable for large scale production (high fitting up costs, low marginal costs). In this case it was the marginal cost function that was discontinuous, such that, again, multistability could occur. To our knowledge, this model was never further studied. As in the kinked demand case, Palander based his study on linear functions. However, we prefer to skip the linear format, using an isoelastic demand function, and non-linear cost functions with built-in capacity limits.

In our model we suppose that each firms has two plants. However, as the firms can either operate each of the plants separately or both in combination dividing production between the plants according to the principle of equal marginal costs, it follows that the firms actually have three cost options. Typically, the option chosen depends on output. Suppose we can classify the plants according to their optimal scale of operation; then at a small output the small scale plant will be chosen. With increasing output the choice will shift to the plant appropriate for larger scale production, and eventually both plants will be used, the combination representing the largest scale of all.

Let us consider the following short-run cost function[1]:

$$C(q) = \begin{cases} 0 & \text{if } q \leqslant 0, \\ as_i + c\dfrac{k_i q}{k_i - q} & \text{if } q > 0 \end{cases} \tag{1}$$

with a, c, k_i, s_i real and positive parameters.

In the first term, s_i represents the 'fitting up' or 'setup' costs incurred each time a plant is put into operation. These may be partially common to all the plants (consider, for instance, electricity costs) with some specific costs to each plant, which increase with its dimension (if $k_i > k_j$, and in this case we should expect that $s_i > s_j$). For convenience we introduce a positive parameter a which is allowed to vary so that all the fitting up costs are simultaneously proportionally varied. The second term represents variable costs, and becomes infinite as $q \to k_i$ (thus leading to a capacity limit k_i). A constant returns to scale production function will not do.[2]

For simplicity's sake we shall consider only the dynamics between two firms (as the generalization to more firms is standard, but not the related dynamic model) characterized by bounded rationality. In particular, we consider the case in which firms maximize only the profit of the first period that follows their choices. Concerning the production of the concurrent firm they adopt *naive* expectations. Also, as stated above, we shall assume that both have the same three production options (a so-called symmetric case, although the asymmetric one is quite similar).

This leads us to the study of a two-dimensional model with discontinuities, having both increasing and decreasing jumps. Some results already known for the continuous case, such as the coexistence of several stable cycles, can be extended to the discontinuous one. The study of local bifurcations cannot be extended, as this mainly relates to border-collision bifurcations. Completely different are also the results in terms of global analysis of the basins' structure. As we shall see, we obtain coexistence of different kinds of stable cycles, and even situations of *no single unstable cycle*, a specific situation that can only occur with discontinuous dynamical systems.

The paper is organized as follows: In Section 2 we introduce the model, considering the simple case of two identical firms (which use identical plants in terms of the capacity limits) and derive the reaction functions of the duopolists. In

[1] As we shall see, the dynamic results of the model are congruent with the choice of a short run cost function.

[2] We should use, for instance, a simplified version of the traditional CES function, as suggested by one of the present authors in several recent publications (see Puu, 2005, 2007, 2008).