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Sequential surrogate modeling for efficient finite element model updating

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ABSTRACT

Despite the numerous studies concerning finite element model updating (FEMU), a challenging computational cost issue persists. Therefore, surrogate modeling has recently gained considerable attention in FEMU. Conventionally, surrogate models are constructed by identical samples for all outputs. It is very inefficient and subjective, if various response-surfaces exhibit even for identical parameters. Accordingly, we propose a sequential surrogate modeling for FEMU. It uses infill criteria to guide sampling for updating surrogate models automatically. The proposed method is successful to construct the different response-surfaces and apply FEMU. It is promising for constructing surrogate models with minimal user intervention and tremendous computational efficiency.

1. Introduction

Considering that current design and assessment procedures do not have any quantitative linkage to actual existing structures [1], a process to associate physical models with corresponding existing structures is necessary for the condition assessment.

Finite element (FE) model updating is a representative of such a process, and is based on the inverse problem of identifying structural parameters by refining an initial FE model based on experimental data. FE model updating can be categorized into deterministic and non-deterministic approaches. In the deterministic approach [2–5], a residual between measured and computed reference properties is used as an objective function, and an iterative optimization scheme is employed to minimize the objective function by adjusting the model parameters; whereas, the non-deterministic approach takes into account the uncertainties associated with modeling and incomplete measurement data [6–11]. This approach involves finding the most probable models based on the measured data, using a Bayesian statistical framework, and interval and gap analysis.

The most important task in FE model updating is to minimize the systematic error in the FE model. Many engineers prefer using simple approaches owing to their computational efficiency, despite the availability of much more sophisticated modeling approaches [1]. Many researchers have noted that such simple modeling approaches are inadequate, because of their inability to accurately simulate the actual behavior of real structures. Such simple modeling approaches may results in the systematic errors due to modeling simplifications [12], the omission of structural components [13], and FE discretization errors [14]. It is obvious that the presence of systematic errors results in bias in the model prediction, and this leads to incorrect estimations of the updating parameter [15]. Depending on the modeling and our experience, a high-fidelity FE model can increase the required computational time from only seconds to minutes for a simple analysis (e.g., modal analysis). For a single run, this would not be demanding. However, if the FE analysis must be iterated many times, the resulting process would be highly computational-resource intensive.

In this context, surrogate models have recently attracted considerable attention as faster alternatives to the iterative FE analyses. Surrogate modeling is a method of emulating a computer simulation model in the form of a mathematical/statistical approximation, using the input and output of an FE analysis. The fundamental concept of applying surrogating model to reliability analysis is not entirely new. However, the use of surrogate models for FE model updating has been investigated recently, especially in the civil engineering community [16]. Some examples of surrogate models that have so far been investigated are Multilayer perception [17], polynomial model [16,18–20], moving least square method for a polynomial model [21]; radial basis function [22]; Kriging model [23]. Surrogate models are constructed by training samples in the parameter space; therefore, generating the samples for the construction of a surrogate model is a key task. Consequently, conventional surrogate modeling for FE model updating has been investigated from a design of experiments (DOE) in the

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previous studies, such as central composite design [16,18,21,22], uniform design [20], D-optimal design [19], and Sobol sequence sampling [23].

The conventional approach generally employs a trial-and-error method based on different designs (i.e., different subsets) of the training samples, because the response-surface is not known beforehand. It is also difficult to represent complicated response-surfaces in the conventional approach under local variations of response behaviors and non-linearity, because the conventional approach generates samples that spread out uniformly across the parameter space. In addition, it is inefficient to apply the identical training samples to all target outputs, if identical updating parameters of the FE model can generate the different response-surfaces of the target outputs due to their relative sensitivity.

To address the abovementioned difficulties, we propose a sequential surrogate (SS) modeling for the efficient FE model updating based on the Kriging model. The proposed method is able to address the abovementioned difficulties of the conventional approach. One crucial advantage of the proposed method is the ability to statistically interpret the uncertainty in the prediction, so that this approach can use the measure of infill criteria and update a surrogate model by adding a new sample.

The rest of this paper is organized as follows. In Section 2, we first describe the mathematical background of the Kriging model, including the statistical interpretation of the Kriging prediction. Next, we present a conventional sequential surrogate modeling originated from the global optimization community [24,25], and a potential problem in FE model updating is discussed. In order to address the potential problem, we propose a sequential surrogate modeling for FE model updating. In Section 3, FE model updating based on the Kriging model with the proposed method is performed numerically and experimentally, using a lab-scaled five-story shear building structure. In addition, the computational efficiency is discussed. In Section 4, we provide concluding remarks on the study.

2. Sequential surrogate modeling based on Kriging model

2.1. Kriging model

The Kriging model is a surrogate model that originated from Geostatistics [26]. The Kriging model is a way of modeling a function as a realization of Gaussian process. Assuming that the function being modeled is continuous, two samples of the true function will tend to have similar values if the distance between the two samples decreases. This spatial correlation can be used to estimate an unknown function value from the known function values. This property can be given the statistical interpretation that the values of the function are correlated with a spatial distance. Therefore, this spatial correlation can be modeled statistically, using the relative distances between the samples. The k-dimensional Kriging basis can be expressed as

$$\psi^{ij} = \exp\left(-\sum_{p=1}^{k}\theta_p\left\|x_p^i - x_p^j\right\|^{m_p}\right) = corr\left[y(\boldsymbol{x}^i), y(\boldsymbol{x}^j)\right] \tag{1}$$

where subscript "p" denotes the dimension of sample x, the superscripts "i" and "j" indicate the ith and jth sample, respectively, and $\left\|x^i - x^j\right\|^{m_p}$ is the relative distance measure between two samples in a parameter space with m-norm. The Kriging basis contains parameters corresponding to each dimension (θ_p and m_p) that determine how fast the correlation decays in each dimension. Therefore, these parameters serve to reflect the significant importance of each dimension [25]. The main purpose of using this flexible basis is to express the various shapes of the spatial correlation. To reduce the computational complexity, Eq. (1) can be expressed as a Euclidean distance ($m_p = 2$). In the remainder of this paper, boldface indicates a matrix or vector.

2.1.1. Modeling of Kriging model

Using a random vector ($\boldsymbol{Y}$), the values of a function of n samples can be represented as

$$\boldsymbol{Y} = \left[y(\boldsymbol{x}^1)y(\boldsymbol{x}^2)\cdots y(\boldsymbol{x}^n)\right]^T \tag{2}$$

This random vector ($\boldsymbol{Y}$) has a mean of $\boldsymbol{1}\mu$, where $\boldsymbol{1}$ is an n-by-1 unit vector, and a variance of σ^2. Assuming that the realizations ($y(\boldsymbol{x})$) of this random vector are correlated, the correlation function can be defined as in Eq. (1), using a 2-norm ($m_p = 2$). Therefore, the correlation matrix of all samples can be constructed as

$$\boldsymbol{\Psi} = corr[\boldsymbol{Y},\boldsymbol{Y}] = \begin{bmatrix} corr\left[y(\boldsymbol{x}^1), y(\boldsymbol{x}^1)\right] & \cdots & corr\left[y(\boldsymbol{x}^1), y(\boldsymbol{x}^n)\right] \\ \vdots & \ddots & \vdots \\ corr\left[y(\boldsymbol{x}^n), y(\boldsymbol{x}^1)\right] & \cdots & corr\left[y(\boldsymbol{x}^n), y(\boldsymbol{x}^n)\right] \end{bmatrix} \tag{3}$$

The covariance matrix can be derived from the correlation matrix ($\boldsymbol{\Psi}$), as

$$COV(\boldsymbol{Y},\boldsymbol{Y}) = \sigma^2\boldsymbol{\Psi} \tag{4}$$

The variance σ^2 determines the overall dispersion relative to the mean of the Kriging model.

The value of the function represents the realization of the Gaussian process, so that μ and σ^2 are estimated using the observed pattern of the training samples.

In a similar manner to statistical estimation theory, the parameters are estimated using maximum likelihood estimation. The likelihood function is given by

$$L = \frac{1}{(2\pi\sigma^2)^{n/2}|\boldsymbol{\Psi}|^{1/2}}\exp\left[-\frac{(\boldsymbol{Y}-\boldsymbol{1}\mu)^T\boldsymbol{\Psi}^{-1}(\boldsymbol{Y}-\boldsymbol{1}\mu)}{2\sigma^2}\right] \tag{5}$$

To simplify the likelihood function with numerical stability, the natural logarithm of Eq. (5) is taken, and the constant term is then ignored. Therefore, we obtain

$$\ln(L) \approx -\frac{n}{2}\ln\left(\sigma^2\right) - \frac{1}{2}\ln|\boldsymbol{\Psi}| - \frac{(\boldsymbol{Y}-\boldsymbol{1}\mu)^T\boldsymbol{\Psi}^{-1}(\boldsymbol{Y}-\boldsymbol{1}\mu)}{2\sigma^2} \tag{6}$$

By taking the derivatives of Eq. (6) with respect to μ and σ^2 and setting these to zero, the maximum likelihood estimators for μ and σ^2 are derived as follows:

$$\hat{\mu} = \frac{\boldsymbol{1}^T\boldsymbol{\Psi}^{-1}\boldsymbol{Y}}{\boldsymbol{1}^T\boldsymbol{\Psi}^{-1}\boldsymbol{1}} \tag{7}$$

$$\hat{\sigma}^2 = \frac{(\boldsymbol{Y}-\boldsymbol{1}\mu)^T\boldsymbol{\Psi}^{-1}(\boldsymbol{Y}-\boldsymbol{1}\mu)}{n} \tag{8}$$

By substituting Eqs. (7) and (8) into Eq. (6) and ignoring the constant terms, we obtain the so-called concentrated log-likelihood function, as

$$\ln(L) \approx -\frac{n}{2}\ln\left(\hat{\sigma}^2\right) - \frac{1}{2}\ln|\boldsymbol{\Psi}| \tag{9}$$

It is not possible to differentiate Eq. (9). However, it is obvious that maximum likelihood estimates ($\hat{\mu}$ and $\hat{\sigma}^2$) can be sequentially computed from the correlation matrix ($\boldsymbol{\Psi}$) and the training samples, so that the only remaining parameter to be determined in Eq. (1) is θ_p. Therefore, an optimization method is applied to find the optimal θ_p by maximizing Eq. (9).