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A statistical equilibrium model of competitive firms

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ABSTRACT

We find that the empirical density of firm profit rates, measured as returns on assets, is markedly non-Gaussian and reasonably well described by an exponential power (or Subbotin) distribution. We start from a statistical equilibrium model that leads to a stationary Subbotin density in the presence of complex interactions among competitive heterogeneous firms. To investigate the dynamics of firm profitability, we construct a diffusion process that has the Subbotin distribution as its stationary probability density. This leads to a phenomenologically inspired interpretation of variations in the shape parameter of the Subbotin distribution, which essentially measures the competitive pressure in and across industries. Our findings have profound implications both for the previous literature on the 'persistence of profits' as well as for understanding competition as a dynamic process. Our main formal finding is that firms' idiosyncratic efforts and the tendency for competition to equalize profit rates are two sides of the same coin, and that a ratio of these two effects ultimately determines the dispersion of the equilibrium distribution.

> Profit is so very fluctuating, that the person who carries on a particular trade, cannot always tell you himself what is the average of his annual profit. It is affected, not only by every variation of price in the commodities which he deals in, *but by the good or bad fortune both of his rivals and of his customers, and by a thousand other accidents* [...] (emphasis added).
>
> Smith (1776, p. 58).

1. Competition and profitability

We propose a statistical equilibrium methodology that accounts for the empirical distribution of firm profit rates, which turns out to be well described by a Laplace distribution. Our findings have profound implications for the specific time evolution of individual firm profitability and the previous 'persistence of profits' literature, and for our understanding of competition as a dynamic and inherently stochastic process, as already envisioned in the introductory quote by Smith.

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The notion of economic competition comes in many forms and varieties, and it is certainly one of the most pervasive concepts in the history of economic thought (see, e.g., Stigler, 1957; Vickers, 1995). Following Cournot, the dominant strand of thought associates (perfect) competition with a particular market form, and emphasizes the efficient allocation of resources at points where prices equal marginal costs (see, e.g., McNulty, 1968). Another important strand of thought originates with Smith's notion of competition as a dynamic process that leads to a tendency for profit rate equalization, which we henceforth label as classical competition.[1] Classical competition essentially describes a negative feedback mechanism. Capital will seek out sectors or industries where the profit rate is higher than the economy-wide average, typically attracting labor, raising output, and reducing prices and profit rates, which in turn provides an incentive for capital to leave the sector, thereby leading to higher prices and profit rates for firms that remain in the sector (see, e.g., Foley, 2006). As a result, classical competition tends to equalize profit rates, yet it simultaneously leads to perpetual changes in technologies and competitive practices. Coupled with continually changing tastes of consumers, and the entry and exit dynamics of rival firms, the very nature of (classical) competition renders a complete elimination of differences in and across sectoral profit rates improbable.

Modeling the process of competition is made all the more difficult by the interactions among firms, which in themselves create a complex environment that feeds back into the destinies of individual companies. One company's gain is often the loss of others, particularly in situations where resources are limited, for instance when it comes to the hiring of exceptional talent, the retainment and acquisition of clients, or the patenting of new technologies. Positive feedbacks, typically arising from symbiotic relationships and synergetic interactions, further increase the complexity of the competitive environment.

The interactions of competitive firms and their idiosyncratic efforts to stay ahead of the game give rise to an enormous amount of information and complexity that is hard to approach from a deterministic viewpoint. In light of the intricate connections and interactions among business firms, our focus shifts accordingly from a fixed-point equilibrium to the notion of a *statistical equilibrium* in the spirit of Foley (1994). Formally, Foley's statistical equilibrium theory of markets revolves around the *maximum entropy principle* (MEP) of Jaynes (1978). After all, MEP derives the combinatorially most likely (or informationally least biased) distribution of a random variate subject to moment constraints. Thus, instead of considering competitive equilibrium as a situation in which all economic agents face an identical profit rate, our statistical equilibrium model stresses the *stationary distribution* of profit rates. While MEP is an intuitive starting point of probabilistic modeling, it cannot shed light on the dynamics underlying the statistical equilibrium distribution of profit rates. Thus we extend the statistical equilibrium concept through the use of a stochastic differential equation, constructing a diffusion process that results in the empirically observed equilibrium distribution of profit rates, and allows for a more disaggregated description of the process of competition.

Approaching the profitability of business firms from a probabilistic perspective is methodologically related to a long-standing tradition that revolves around distributional regularities in a wide range of other socio-economic variables (see, e.g., Champernowne, 1953; Gibrat, 1931; Kalecki, 1945; Pareto, 1897; Simon, 1955; Steindl, 1965). But these approaches start from very disaggregated and thus quite specific stochastic processes, while our formulation of competition as a statistical equilibrium outcome concerns a more aggregated level of stochastic modeling.

In order to apply the maximum entropy formalism to any kind of economic phenomenon, one essentially needs to encode the economic contents in terms of moment constraints (see, e.g., Castaldi and Milaković, 2007; Foley, 1994; Stutzer, 1996). Hence, modeling classical competition by way of MEP boils down to expressing competition in the form of moment constraints. We take the position that the average profit rate corresponds to a measure of central tendency, while the complex movements of capital in search of profit rate equalization and the resulting feedback mechanisms translate into a generic measure of dispersion around the average. When the number of competitive firms in a decentralized type of market organization is large, probabilistic factors can give rise to statistical regularities in the distribution of profit rates. The distribution of profit rates that can be achieved in the most evenly distributed number of ways under the dispersion constraint is then the statistical equilibrium or maximum entropy distribution of profit rates, and turns out to be an *exponential power* or *Subbotin* distribution.

The Subbotin (1923) distribution has three parameters: a location, a scale, and a shape parameter. Structural differences in the statistical equilibrium model stem from differences in the shape parameter, because operating on the location or scale parameter does not change the qualitative features of the Subbotin distribution. If the shape parameter is equal to two, the Subbotin distribution reduces to the Gaussian (normal) distribution, and if it is equal to unity, the Subbotin distribution reduces to the Laplace (double-exponential) distribution.

To demonstrate the empirical relevance of our model, we investigate the empirical density of profit rates, which is indeed reasonably described by a Laplace distribution. This prompts us to ask why the empirical shape parameter is close to unity, what this implies about the competitive environment that firms are facing, and whether variations in the shape parameter correspond to qualitative changes in the competitive environment. Since the maximum entropy principle only informs us of the equilibrium distribution, it cannot describe the dynamics that lead to the equilibrium distribution. In order to extend the model in a dynamic direction, we utilize a particular class of stochastic processes known as *diffusion*

[1] Schumpeter's theory of innovation and creative destruction, or evolutionary theories of industrial dynamics (see, e.g., the edited volume by Dosi et al., 2000), also highlight the intrinsically dynamic character of economic competition and would be consistent with the notion of 'classical' competition from this viewpoint.