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Vibrations of cracked beams: Discrete mass and stiffness models

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ABSTRACT

The present work aims at contributing to the characterization of the nonlinear dynamic behavior of structures incorporating cracks. With this purpose, the Discrete Element Method is adopted in conjunction with an existing result from fracture mechanics that takes into account the local flexibility of a cracked beam. The dynamic behavior of a cantilever beam and of a beam free of support conditions is studied and the effect of the presence of a crack is analyzed. When possible, the results are compared with exact solutions or with experimental data from the literature.

1. Introduction

The detection of cracks in structures is a very important issue in many engineering branches such as Civil, Mechanical and Aeronautical and more practical and less onerous detection methods allowing to assess the location and depth of the cracks are continuously investigated. The dynamics of cracked beams has been studied extensively in recent years both experimentally and theoretically using analytical, semi-analytical or numerical methods. In some studies, the presence of cracks is modeled by the introduction of linear springs into the undamaged structure at the location of the cracks [1–5]. Bilinear springs were used to model cracks that open and close during the oscillation (breathing cracks) [6,7]. Linear and nonlinear finite element analyses using specific finite elements for the section of the crack based on the strain energy function given by the linear fracture mechanics theory have also been performed [8–13]. Single–degree–of–freedom models [14–16] were also used to simulate the nonlinear behavior of beams with breathing cracks. In [17], a continuous modification of the stress field in the beam induced by a crack was taken into account by an experimental function exponentially decaying with the distance from the crack. A simplified (linear) variation of the bending stiffness in the vicinity of the crack was considered in [18]. A continuous cracked beam model consistent with linear fracture mechanics was suggested in [19]. We also refer to [20] for a comprehensive review on several approaches for the modeling of cracks.

This work intends to contribute to the characterization of the free or forced vibrational response of some structures incorporating cracks of different characteristics. The method used in the development of the models presented in this paper is the Discrete Element Method (DEM) [21]. Using this method the beam is represented as a discrete system of blocks (i.e. with a finite number of degrees of freedom), where the mass and the moment of inertia of each block are lumped in its middle point, linked by rotational and transverse springs that simulate the bending and shear compliances, respectively. Once the model is defined, the differential equations governing its time evolution are established. For the particular case of a beam, these equations involve relative rotations and displacements between blocks, as well as their derivatives with respect to time. One of the most important criteria to obtain good results using the DEM is the process adopted for the beam discretization. It is expectable that, as one refines the mesh, the results tend towards the exact solutions. However, the number of blocks should not be indefinitely increased as that would lead to an increase of the time expended in the numerical calculations; there should exist a balance between the time and the effort used in the calculations and the aimed precision for the results. The easiness with which the DEM considers cases where the beams are cracked should be emphasized. Assuming the existence of cracks between rigid blocks, the localized loss of stiffness (coincident with the crack position) is taken into consideration in this

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work by changing the stiffness of the springs connecting the blocks; the amount of reduction of the stiffness will be dependent on the crack depth according to Okamura et al. [22]. The expression for the local flexibility of a cracked beam was derived in [22] for the computation of the load carrying capacity of a cracked column, using linear fracture mechanics. To the best of our knowledge this is the first time that the effect of a crack on the local flexibility of a beam as derived in [22] is used together with the DEM [21] to study the dynamic behavior of cracked beams, although the rigid finite element method [23], which has features in common with the DEM, has already been used to study the vibration of a cracked rotor [13].

In this work, the dynamic behavior of several beam models is studied and simulated with the help of the DEM implemented in Matlab environment [24]. In Section 2 a cantilever beam model is studied and the system of ordinary differential equations that governs its motion is obtained. The existence of a breathing crack (a crack that opens and closes accordingly to the sign of the curvature of the cross section) is also considered. The time integration of the equations that govern the motion of a cracked cantilever beam is performed and the time evolution of the beam's dynamic response is presented and compared with experimental results. In Section 3, an analogous study is made for a suspended beam submitted to an oscillatory external force acting perpendicularly to the beam's suspension plane and free of support conditions in the forcing direction. The last section is dedicated to the conclusions and to the enumeration of some aspects that are worth of future attention.

2. Cantilever beam

2.1. Dynamics of a homogeneous cantilever beam

In this section a cantilever beam of length L with uniform rectangular cross section $b \times h$, mass density ρ and subjected at its free end to an external concentrated time varying force is considered (Fig. 1a). The beam is decomposed in N blocks and each pair of consecutive blocks is connected by a pair of springs (rotational and transverse) (Fig. 1b). The first and last blocks have a length that is half the length of the intermediate blocks, the latter with length $l = \frac{L}{N-1}$, mass $m = \rho bhl$ and moment of inertia $J = \frac{\rho bhl}{2}(h^2 + l^2)$ around an axis perpendicular to the plan of motion. The mass and moment of inertia of the first and last blocks are $m_e = \rho bh\frac{l_n}{2}$ and $J_e = \frac{\rho bh(l_n/2)}{12}\left(h^2 + \left(\frac{l_n}{2}\right)^2\right)$, respectively. The first block is considered to be clamped.

Applying d'Alembert's principle to the n-th block (Fig. 2) we obtain the equations that govern its translational and rotational motion

$$m_n \ddot{y}_n = S_{n+1} - S_n, \quad n = 2, \ldots, N-1, \tag{1}$$

$$J_n \ddot{\theta}_n = M_{n+1} - M_n + \frac{l_n}{2}(S_{n+1} + S_n), \quad n = 2, \ldots, N-1. \tag{2}$$

Applying d'Alembert's principle to the last block, where the system of external forces was reduced to its center of mass (Fig. 3), we get

$$m_N \ddot{y}_N = -S_N - F(t), \tag{3}$$

$$J_N \ddot{\theta}_N = -M_N + \frac{l_N}{2}S_N - \frac{l_N}{2}F(t). \tag{4}$$

Eqs. (1)–(4) may be written in matrix form as

$$\mathbf{m}\ddot{\mathbf{y}} = \mathbf{AS} - F(t)\mathbf{e}_{N-1}, \tag{5}$$

$$\mathbf{J}\ddot{\boldsymbol{\theta}} = \mathbf{AM} + \frac{1}{2}\mathbf{LBS} - \frac{l_N}{2}F(t)\mathbf{e}_{N-1}, \tag{6}$$

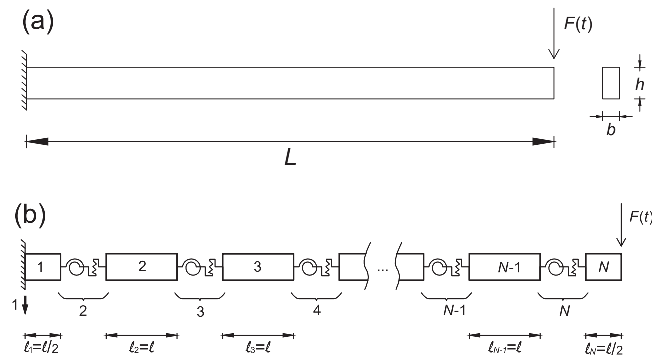

Fig. 1. (a) Homogeneous cantilever beam with rectangular cross section. (b) Discrete element model of the cantilever beam where the stiffness is discretized at the interfaces between blocks.

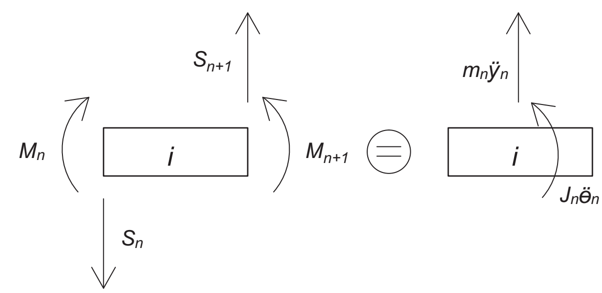

Fig. 2. Model of the n-th block of the cantilever beam.

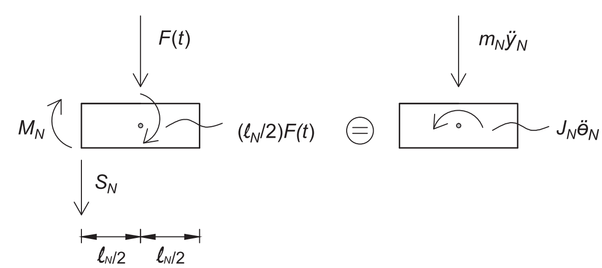

Fig. 3. Model of the last block of the cantilever beam subjected to an external transverse concentrated force at the tip.

where $\ddot{\mathbf{y}} = \{\ddot{y}_2\ \ddot{y}_3 \cdots \ddot{y}_N\}^T \in \mathbb{R}^{N-1}$, $\ddot{\boldsymbol{\theta}} = \left\{\ddot{\theta}_2\ \ddot{\theta}_3 \cdots \ddot{\theta}_N\right\}^T \in \mathbb{R}^{N-1}$, $\mathbf{S} = \{S_2\ S_3 \cdots S_N\}^T \in \mathbb{R}^{N-1}$, $\mathbf{M} = \{M_2\ M_3 \cdots M_N\}^T \in \mathbb{R}^{N-1}$, $\mathbf{e}_{N-1} = \{0 \cdots 0\ 1\}^T \in \mathbb{R}^{N-1}$, $\mathbf{m} = \mathrm{diag}(m_2, m_3, \ldots, m_N) \in \mathbb{R}^{(N-1)\times(N-1)}$, $\mathbf{J} = \mathrm{diag}(J_2, J_3, \ldots, J_N) \in \mathbb{R}^{(N-1)\times(N-1)}$, $\mathbf{L} = \mathrm{diag}(l_2, l_3, \ldots, l_N) \in \mathbb{R}^{(N-1)\times(N-1)}$,

$$\mathbf{A} = \begin{bmatrix} -1 & 1 & 0 & \cdots & & 0 \\ 0 & -1 & 1 & & & \vdots \\ \vdots & & & & & \\ & & & 0 & -1 & 1 \\ 0 & \cdots & & & 0 & -1 \end{bmatrix} \in \mathbb{R}^{(N-1)\times(N-1)} \tag{7}$$

and

$$\mathbf{B} = \begin{bmatrix} 1 & 1 & 0 & \cdots & & 0 \\ 0 & -1 & 1 & & & \vdots \\ \vdots & & & & & \\ & & & 0 & 1 & 1 \\ 0 & \cdots & & & 0 & 1 \end{bmatrix} \in \mathbb{R}^{(N-1)\times(N-1)}. \tag{8}$$