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Mixing dynamic relaxation method with load factor and displacement increments

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ABSTRACT

Dynamic Relaxation (DR) method is an explicit iterative technique suitable for nonlinear structural analysis. In this method, the static equilibrium equations are converted to a fictitious dynamic system. In general, DR iterations are unstable. Nevertheless, the fictitious parameters, such as the diagonal mass and damping matrices as well as the time steps are determined so that the stability conditions are satisfied. If the mass and damping matrices are selected properly, the structural responses converge to the accurate static solutions. Nonlinear behavior of structures includes various characteristics, such as the existence of load limit points; displacement limit points, buckling points, post-buckling region and bifurcation points. All of these valuable features are demonstrated by the equilibrium path, and most of the DR procedures cannot completely trace it. To overcome this weakness, a new variable load factor is presented by minimizing the unbalanced displacement. In the second suggested algorithm, the load factor is determined based on the parts of the structural equilibrium path placed between two limit points. In order to prove the capability of the proposed strategies, several 3D trusses and 2D frames, with geometrical nonlinear behavior, are analyzed. The numerical results indicate that the proposed approach can trace the complex structural equilibrium path.

1. Introduction

In the structural nonlinear analysis, the internal forces or stiffness matrix entries are functions of nodal displacements. The responses of structure can be obtained mostly with the help of iterative techniques. Usually, the direct methods and the iterative schemes should be mixed to achieve the correct nonlinear solutions. Equilibrium path can illustrate nonlinear structural behavior efficiently. So far, various methods for tracing the equilibrium path of structures have been presented. Hellweg and Crisfield proposed a new strategy for choosing the correct root in the cylindrically arc length method [1]. The stability analysis of 3D trusses was performed by Greco and Venturin [2]. They found a new formulation for geometric nonlinear analysis of structures. This approach was based on the nodal positions, instead of the nodal displacements. The variable arc length method was used for nonlinear analysis of structures by Rezaiee-Pajand and Boroshaki [3]. In 2008, the modified normal flow procedure was suggested by Saffari et al. They studied geometric nonlinear behavior of Space Trusses [4]. Furthermore, Saffari and Mansouri recommended the two-point procedure for solving the governing equilibrium equations of structures [5]. This algorithm was not able to pass the load limit points. Some other researchers also studied the tracing of structural equilibrium path [6–14].

Frequently, the iterative methods are divided into two categories: 1 – explicit techniques 2 – implicit techniques. The first group uses vector operators. This is because these methods deploy internal forces to achieve the responses. The simplicity and high efficiency of the explicit approaches are their main characteristics. In implicit procedures, the load and displacement limit points cause difficulties in the solution procedure. It is worth emphasizing; the implicit methods are more complex and time-consuming than the explicit ones. This is rooted in the fact that the implicit schemes employ matrix operators. However, the convergence rate of the implicit methods is better, in comparison to the explicit ones [15].

It should be mentioned that the DR methods are placed into the group of the explicit process. In this method, the fictitious mass and the fictitious damping are added to the static governing equations, and therefore, an artificial dynamic system is constructed. According to Lee et al. findings: "The mass matrix needs not to be the true structural mass, because the DRM is not for the dynamic problem, but it is for the quasistatic or static one. The fictitious mass, which is proportional to the stiffness, is sufficient for

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Nomenclature

C	damping matrix	δX	displacement increment
e	convergence criteria	$\dot{X}$	velocity
F, f	internal forces	ω	frequency
$\dot{f}$	internal force increment	λ	load factor
K	stiffness matrix	$\delta\lambda$	load factor increment
P	external forces	i, j	numerator of degrees of freedom
R, r	residual forces	n	iteration number
M	mass matrix	ndof	number of degrees of freedom
t	fictitious time		
X	displacement		

enhancing the numerical stability and increasing the convergence speed [16].” In other words, the response of the structure can be achieved under the condition that the vectors related to the inertia and damping forces are equal to zero.

For the first time, Otter and Day used this technique in 1960s [17–19]. DR approach was based on the Richardson’s second order strategy developed by Frankel [20]. Before the others, Rushton utilized this algorithm for the geometric nonlinear analysis of bending plates [21]. This method was applied for form-finding the stable condition and the analysis of both nets and prestressed membrane roofs by Barnes [22]. This researcher also investigated the tension structures and their stable condition [23,24]. Han and Lee studied the stabilizing process of unstable structures by employing the dynamic relaxation method [25].

In this paper, a new relation for DR technique capable of tracing the equilibrium path is presented. This algorithm deploys the load factor increment and the minimization of the displacement increment, simultaneously. In this way, the displacement increment will be established based on the residual force and load factor increments. By setting its first-order derivative with respect to the load factor increment equal to zero, a new formula will be found. As a result, the proposed load factor depends only on the DR fictitious parameters.

This article includes the following sections. At the first stage, the basis of dynamic relaxation approach and the corresponding formulations are discussed. Then, the other researchers’ methods for tracing the structural equilibrium path are reviewed. Afterward, a new strategy for specifying the load factor is suggested. Moreover, in another scheme, the new method and other researchers’ procedures are mixed. Finally, several numerical studies are performed to investigate the capability of the recommended techniques.

2. Dynamic relaxation method

In DR approach, the fictitious mass and the fictitious damping are added to the structural static equations. Through this method, an artificial dynamic system of equations is achieved. In each time step of this procedure, it is presumed that the velocity varies linearly and the acceleration remains constant. On the other hand, the diagonal mass and the diagonal damping matrices are selected. Based on these assumptions, the dynamic relaxation method is considered as an explicit technique and uses vector operators for the analysis. With the help of the central finite difference scheme, the iterative equations of the aforesaid method can be obtained in the below form [26]:

$$\dot{X}_i^{n+\frac{1}{2}} = \frac{2m_{ii}^n - C_{ii}^n t^n}{2m_{ii}^n + C_{ii}^n t^n}\dot{X}_i^{n-\frac{1}{2}} + \frac{2t^n}{2m_{ii}^n + C_{ii}^n t^n} r_i^n, \quad i = 1, 2, \ldots, ndof \tag{1}$$

$$X_i^{n+1} = X_i^n + \delta X_i^n = X_i^n + t^{n+1}\dot{X}_i^{n+\frac{1}{2}}, \quad i = 1, 2, \ldots, ndof \tag{2}$$

Herein, the i-th diagonal elements of the fictitious mass and damping matrices, time step and the i-th element of the residual force vector are denoted by m_{ii}^n, C_{ii}^n, t^n and r_i^n, respectively. Note that the superscript n is the indicator of the n-th iteration. Additionally, the number of degrees of freedom is shown by $ndof$. The displacement increment is denoted by δX. Moreover, X and $\dot{X}$ are the displacement and velocity vectors, respectively. So far, numerous researches have been conducted to study the fictitious parameters of the DR method [26–34]. They demonstrated the abilities of their formulas by solving some numerical examples. The following paragraphs are devoted to briefly introduce these works for the last two decades.

The nodal damping was suggested by Zhang et al. [35]. On the other hand, Munjiza et al. expressed the damping in terms of the exponent of the mass and stiffness matrices [36,37]. Rezaiee-Pajand and Taghavian Hakkak calculated the displacement by utilizing the first three terms of the Taylor series [38]. Based on the minimization of the residual forces, the fictitious time step was formulated by Kadkhodayan et al. [39]. In another paper, Rezaiee-Pajand and Alamatian performed a nonlinear dynamic analysis by using DR method [40]. This algorithm reduced the integration errors.

In 2010, Rezaiee-Pajand and Sarafrazi proposed new formulas for finding the optimum time step and the critical damping [41]. According to this study, it is proved that the value of the constant time step does not affect the convergence rate. The formulas for artificial mass and damping were recommended by Rezaiee-Pajand and Alamatian [42]. Additionally, based on the error minimization between two successive steps, another relation was proposed for the calculate of the fictitious damping [43]. By assuming that damping is equal to zero and calculating the relevant time step ratio, Rezaiee-Pajand and Sarafrazi suggested another method [44]. Moreover, a new technique was obtained, based on the out-of-balance energy [45]. In an extended study, Rezaiee-Pajand et al. investigated the efficiency of the twelve well-known DR approaches in the finite element analysis of frames and trusses [46].

In the coming sections, the characteristics of the DR strategy are mentioned. In this technique, the internal forces of each element are achieved by using the products of the nodal displacements and the element stiffness matrix. By assembling the elemental internal forces, the internal force vector of the structure is gained. The residual force vector is defined as the difference between the internal and external force vectors. Hence, this scheme is appropriate for solving the problems with extremely nonlinear geometric and material behaviors. It is worth emphasizing that instead of complex matrix computations, utilizing the vector operator reduces the required computer storage memory. Since the DR