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Analytic solving of asset pricing models: The by force of habit case

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ABSTRACT

Analytic methods for solving asset pricing models are developed to solve asset pricing models. Campbell and Cochrane's [1999. By force of habit, a consumption-based explanation of aggregate stock market behavior. Journal of Political Economy 107, 205–251] habit persistence model provides a prototypical example to illustrate this method. When the parameters involved satisfy certain conditions, the integral equation of this model has a solution in the space of continuous functions that grows exponentially at infinity. However, the parameters advocated by Campbell and Cochrane do not satisfy one of these conditions. The existence problem is removed by restricting the price–dividend function to avoid values of dividend growth that are extreme. Thus, existence and uniqueness of the solution in the space of continuous and bounded functions is proved. Using complex analysis the price–dividend function is also shown to be analytic in a region large enough to cover all relevant values of dividend growth. Next, a numerical method is presented for computing higher order polynomial approximations of the solution. Finally, a uniform upper bound on the error of these approximations is derived. An intensive search of the parameter space results in no parameter values for which the solution matches the historic equity premium and Sharpe ratio within Campbell and Cochrane's model.

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1. Introduction

Various asset pricing models have been proposed to explain the equity premium puzzle, i.e., the high return on stocks relative to the return on risk-free bonds.[1] Each of these models produces an integral equation whose solution determines the price–dividend function and hence also the equilibrium return on a risky asset held by a representative investor. Although there is limited information about the essential properties of the solutions to these integral equations, economists and mathematicians employ numerical methods to approximate them. The recent work of CCCH and CCH shows how to obtain the essential mathematical properties of the solutions to the well-known asset pricing models of Mehra and Prescott (1985) and a generalized version of Abel (1990).[2] They go on to show how these properties can lead to improvements in the numerical algorithms developed to represent the solutions to these models. This paper lays out how these analytic methods can be used to identify the mathematical properties of more general asset pricing problems. Subsequently, it shows how to develop numerical algorithms exploiting these properties.

To illustrate these analytic methods we use the asset pricing model of Campbell and Cochrane (CC) (1999). A shortcoming of early asset pricing models is that they require unrealistically large levels of relative risk aversion and/or high risk free interest rates in order for their price–dividend functions to yield equity premiums consistent with empirical evidence.[3] CC seek to reconcile this gap between theory and evidence by introducing a surplus consumption ratio into the pricing kernel. This surplus consumption ratio compares the investor's consumption with their external habit. They also develop an equation of motion for this surplus consumption ratio, which is dependent on random shocks to dividend growth. In this equation of motion the log-normally distributed random shocks to dividend growth are amplified by a sensitivity function that places larger (smaller) weight on small (large) random shocks to dividend growth. This sensitivity function is dependent on the surplus consumption ratio so that the surplus consumption ratio is the state variable for the determination of the price–dividend ratio. This specification creates a precautionary savings which keeps the risk free interest rate low.[4]

The CC model leads to an integral equation for the price–dividend function which depends on the surplus consumption ratio. Moreover, the surplus consumption ratio depends on future dividend growth. In this model it is assumed that dividend growth follows a log-normal distribution. Therefore, it is natural to seek a solution to the integral equation in the space of continuous functions that grow exponentially. That is, the real vector space $C(\mathbb{R}, e^{\gamma x})$ of all continuous functions $f(x)$ with domain $\mathbb{R}$ such that $|f(x)| \leqslant m_1 e^{\gamma x} + m_2$ for all x, where $m_1, m_2 \geqslant 0$ may depend on $f(x)$. When the parameters involved satisfy certain conditions, it is shown that the integral equation of the CC model has a unique solution in the space $C(\mathbb{R}, e^{\gamma x})$ (see Proposition 1).

Unfortunately, the condition for the existence of a unique solution to Campbell and Cochrane's model in the space $C(\mathbb{R}, e^{\gamma x})$ is not satisfied. Given the procedure for choosing the parameter values espoused by CC, it is shown that the coefficient of risk aversion must be larger than 76 for existence of the solution, which is even higher than the value Mehra and Prescott (1985) needed to explain the equity premium. The reason for this failure is that dividend growth is too low relative to the risk free interest rate. Thus, there is no known proof of existence for the original specification of the model by Campbell and Cochrane.

To deal with the existence problem the price–dividend function is restricted to avoid values of the surplus consumption ratio that are extreme. Then, existence and uniqueness of the solution is proved in the space of continuous and bounded functions for a larger set of the parameters, including the ones used by CC. Also, a uniform upper bound for the solution (see Theorem 1) is found. This

[1] See Mehra and Prescott (2003) for a recent survey of these models.

[2] Throughout the paper, we will write CCCH for the paper by Calin et al. (2005), write CCH for the paper by Chen et al. (2008), and write CC for the paper by Campbell and Cochrane (1999).

[3] Weil (1989) refers to this phenomenon as the risk free rate puzzle.

[4] Cecchetti et al. (2000) provide an alternative motive for precautionary savings to explain the equity premium with a low risk free interest rate.