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Properties of equilibrium asset prices under alternative learning schemes

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Abstract

This paper characterizes equilibrium asset prices under adaptive, rational and Bayesian learning schemes in a model where dividends evolve on a binomial lattice. The properties of equilibrium stock and bond prices under learning are shown to differ significantly. Learning causes the discount factor and risk-neutral probability measure to become path-dependent and introduces serial correlation and volatility clustering in stock returns. We also derive conditions under which the expected value and volatility of stock prices will be higher under learning than under full information. Finally, we investigate restrictions on prior beliefs under which Bayesian and rational learning lead to identical prices and show how the results can be generalized to more complex settings where dividends follow either multi-state i.i.d. distributions or multi-state Markov chains.

JEL classification: G12; D83

Keywords: Rational learning; Adaptive learning; Bayesian updating; Lattice models; Asset prices

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1. Introduction

Recent studies have recognized the importance of explicitly incorporating learning effects in equilibrium asset pricing models.[1] Learning introduces a link between state variables and agents' beliefs which the standard assumption of full information rational expectations ignores. This link creates rich dynamics in the mapping from state variables to agents' decisions and thus affects market outcomes such as prices and returns. However, although many alternative learning schemes have appeared in the literature (e.g., adaptive boundedly rational, Bayesian or rational), little is known about their properties when applied to equilibrium asset pricing problems. In fact, the majority of the literature on asset pricing under learning has been developed in a partial equilibrium setting while general equilibrium effects have not received nearly as much attention.[2]

In this paper we show that equilibrium stock and bond prices strongly depend on the nature of the underlying learning process. Our analysis proceeds in the context of one of the cornerstones of modern finance, namely the binomial lattice model proposed by Cox et al. (1979). This model is the discrete time equivalent of the geometric Brownian motion process underlying the Black–Scholes model and has thus been used extensively in finance (see, e.g., Stapleton and Subrahmanyam, 1984). While in the classical finance literature asset prices are assumed to follow a binomial lattice, we assume instead that dividends follow a binomial lattice with unknown probability of an up move, π. In equilibrium, asset prices are determined endogenously as a function of the evolution in agents' beliefs and in dividends.

Existing studies can usefully be separated according to whether they use boundedly rational (adaptive) or fully rational learning (RL) schemes and whether agents use Bayesian or non-Bayesian approaches. Along these lines we compare three learning models, namely Bayesian, rational and adaptive schemes. Bayesian agents view π as a random variable and start with a set of prior beliefs on the probability distribution of π that are updated through Bayes' rule as new dividend information arrives. Under the two other learning schemes, π is viewed as non-random. The adaptive learning (AL) model ignores changes in future parameter estimates, $\hat{\pi}_{t+k}$, viewed from the present (time t), conditioning instead on the current estimate, $\hat{\pi}_t$.[3] In contrast, the forward-looking, 'rational' learning scheme accounts for future updates in $\hat{\pi}_t$, acknowledging that although π is constant, the estimator,

[1] Brennan and Xia (2001), Bullard and Duffy (2001), Timmermann (1993, 1996, 2001), and Veronesi (1999) are among the contributions on the topic.

[2] For instance, Lakner (1995) investigates consumption and portfolio choice in a finite horizon model in which agents have power utility. While asset prices are observable, their drift and the price shocks are not. As a special case, Lakner studies the case where agents learn by recursive application of Bayes' rule and derives the optimal portfolio policy using martingale methods.

[3] Most of the early literature on asset pricing implication of learning adopted the adaptive, least-squares learning approach, see e.g. Timmermann (1993), Barsky and De Long (1993), and Barucci (2000). Sargent (1993) contains a number of applications of boundedly RL schemes to finance. Evans and Honkapohja (1995), Kuan and White (1994), and Marcet and Sargent (1989) proved convergence for parametric least-squares estimators while Chen and White (1998) considered non-parametric estimators that approximate unknown equilibrium relationships with flexible functions.