An interior-point algorithm for computing equilibria in economies with incomplete asset markets

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Abstract

Computing equilibria in general equilibria models with incomplete asset (GEI) markets is technically difficult. The standard numerical methods for computing these equilibria are based on homotopy methods. Despite recent advances in computational economics, much more can be done to enlarge the catalog of techniques for computing GEI equilibria. This paper presents an interior-point algorithm that exploits the special structure of GEI markets. It is proved that, under mild conditions, the algorithm converges globally at a quadratic rate, rendering it particularly effective in solving large-scale GEI economies. To illustrate its performance, relevant examples of GEI markets are solved.
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1. Introduction

Since the time of Radner's (1972) paper, the general equilibrium analysis of sequential economies with incomplete markets has been the subject of extensive

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research in economic theory. These models, known as *general equilibrium with incomplete markets* (GEI), represent the most general and versatile tool in competitive economic theory, as they deal simultaneously with all real and financial markets and their interactions. The size and complexity of these models often demands the use of numerical methods to compute their equilibria. Surveys in this area are due to Magill and Shafer (1991), Magill and Quinzii (1996) and Hens (1998).

The literature on the computation of GEI equilibria is based on *path following* or *homotopy methods*. There are three main approaches to compute equilibria in the GEI model. The first method was given by DeMarzo and Eaves (1996), who considered the excess demand function defined on prices and elements of the Grassmannian manifold (see e.g. Duffie and Shafer, 1985). By applying the work of Brown et al. (1996a), they computed equilibria via a homotopy algorithm. The second algorithm for computing fixed points was developed by Brown et al. (1996b), who considered the excess demand function to be a function of prices only. Because this excess demand function is discontinuous, Brown et al. introduced an auxiliary asset to define a family of homotopies. The third approach is based on the work of Schmedders (1998, 1999), who computed equilibria with homotopy techniques using the first-order conditions of the no-arbitrage agents' problems. To avoid discontinuities in the excess demand correspondence, he considered one agent with penalties for transactions on the asset markets instead of assuming lower bounds on short sales. By making these penalties smaller and smaller, the solutions of the homotopy function move closer and closer to the GEI equilibrium. Other contributions of note are those of Kubler and Schmedders (2000), Kubler (2001) and Herings and Kubler (2002).

Homotopy methods possess powerful theoretical properties, but these methods may be inappropriate from a practical point of view, due to the difficulty of setting up the homotopy function (see Kubler, 2001) and dealing with inequalities (see Watson, 2000). Furthermore, homotopy methods may fail to produce a solution even for relatively simple systems of nonlinear equations (e.g.. see Nocedal and Wright, 1999, Example 11.2). The focus of this paper is a proposal for an alternative algorithm for problems of practical relevance that cannot be solved by the existing approaches.

This paper introduces a tailor-made interior-point algorithm to compute equilibria in economies with incomplete asset markets. I consider a set of optimality conditions which are necessary and sufficient conditions for the existence of a GEI equilibrium, assuming the standard convexity assumptions for the agents' problem. These optimality conditions are the Karush–Kuhn–Tucker (KKT) first-order conditions of the agents' utility maximization problems, the market clearing and no-arbitrage conditions. A distinctive characteristic in solving these optimality conditions is that they often include inequalities (e.g. equilibria can be restricted-domain), which can be difficult to handle using homotopy or continuation methods. I propose an alternative interior-point approach which is tailored to deal effectively with inequality-constrained nonlinear systems of equations.

Although interior-point methods are closely related to central path continuation methods (see Gill et al., 1986), the solution procedures are completely different.