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ELSEVIER Journal of Economic Dynamics & Control 32 (2008) 779–820
JOURNAL OF Economic Dynamics & Control
www.elsevier.com/locate/jedc

Active portfolio management with benchmarking: Adding a value-at-risk constraint

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Received 25 February 2005; accepted 21 March 2007
Available online 14 April 2007

Abstract

We examine the impact of adding a value-at-risk (VaR) constraint to the problem of an active manager who seeks to outperform a benchmark while minimizing tracking error variance (TEV) by using the model of Roll [1992. A mean/variance analysis of tracking error. Journal of Portfolio Management 18, 13–22]. We obtain three main results. First, portfolios on the constrained mean-TEV boundary still exhibit three-fund separation, but the weights of the three funds when the constraint binds differ from those in Roll's model. Second, the constraint mitigates the problem that when an active manager seeks to outperform a benchmark using the mean-TEV model, he or she selects an inefficient portfolio. Finally, when short sales are disallowed, the extent to which the constraint reduces the optimal portfolio's efficiency loss can still be notable but is smaller than when short sales are allowed.

JEL classification: G11; D81

Keywords: Benchmarking; VaR; Tracking error; Portfolio choice; Risk management

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doi:10.1016/j.jedc.2007.03.005

1. Introduction

As Roll (1992) and Cornell and Roll (2005) note, institutional investors often manage money against a benchmark. This has led to the practice by active portfolio managers (hereafter 'managers') of seeking to outperform the benchmark by a given percentage, subject to a limit on *tracking error variance*, or TEV.[1] However, this practice leads such managers to select portfolios that are mean-variance inefficient and under certain conditions have systematic risk that is greater than 1 when measured against the benchmark. Not surprisingly, large losses relative to the benchmark have occurred in some cases. A recent example involved the management of Unilever's pension fund by Merrill Lynch, who in attempting to beat the FTSE All-Share Index by 1% per year, ended up as the defendant in a lawsuit due to underperforming the index by roughly 10% over a 15-month period.[2]

Two methods have been proposed for overcoming this tendency to invest in overly risky portfolios. Roll (1992) advocates constraining the portfolio's beta, while Jorion (2003) advocates constraining the portfolio's variance. In this paper we propose a third method that involves constraining the portfolio's *Value-at-Risk*, or VaR.[3]

A VaR constraint is of particular interest for several reasons. First, as Jorion (2001, 2003) and Pearson (2002) note, the fund management industry is increasingly using VaR to: (1) allocate assets among managers, (2) set risk limits, and (3) monitor asset allocations and managers (these activities are often referred to as 'risk budgeting'). Second, we show that Jorion's result of bringing the optimal portfolio closer to the mean-variance efficient frontier with a variance constraint can also be obtained with a VaR constraint. Third, under certain conditions, the use of VaR as a risk measure is consistent with expected utility maximization (see Alexander and Baptista, 2002). Finally, VaR can be useful to reduce the regret of losses (see Shefrin, 2000).

We begin by examining the case when short sales are allowed. The set of portfolios that minimize TEV for various levels of expected return is referred to as the *mean-TEV boundary*, while the set of portfolios that do so given a VaR constraint is referred to as the *constrained mean-TEV boundary*. Like portfolios on the mean-TEV boundary, we find that portfolios on the constrained mean-TEV boundary exhibit three-fund separation, but the weights of the three funds when the constraint binds differ from those in its absence. Under certain conditions, we find that the constrained mean-TEV boundary consists of: (i) portfolios on the mean-variance boundary, (ii) portfolios on the mean-TEV boundary, and (iii) portfolios that do not belong to either of these boundaries. There are also conditions under which no portfolio on the mean-TEV boundary belongs to the constrained

[1]A portfolio's TEV is the variance of the difference between the returns on the portfolio and the benchmark.

[2]Merrill Lynch agreed to pay $105 million to settle the case. A similar contract existed between Merrill Lynch and Sainsbury that also led to large losses. See *The Wall Street Journal*, December 7, 2001, p. C1.

[3]A portfolio's VaR is the maximum loss at a given confidence level that the portfolio suffers over a time period.