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Discrete time Wishart term structure models

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ABSTRACT

This paper reveals that the class of Affine Term Structure Models (ATSMs) introduced by Duffie and Kan (1996) is larger than previously considered in the literature. In the framework of risk factors following a Wishart autoregressive process, we define the Wishart Term Structure Model (WTSM) as an extension of a subclass of Quadratic Term Structure Models (QTSMs), derive simple parameter restrictions that ensure positive bond yields at all maturities, and observe that the usual constraint on affine processes requiring that the volatility matrix be diagonal up to a path independent linear invertible transformation can be considerably relaxed.

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1. Introduction

A term structure model requires a coherent specification of both the historical and risk-neutral properties of interest rates. While the risk-neutral distribution is used to determine the term structure pattern and compare the current prices of interest rate derivatives, the historical distribution is needed to predict the future term structures, the future derivative prices and to determine the Value-at-Risk of a portfolio of bonds. The most popular specification proposed in the literature and satisfying these requirements is the affine term structure model (ATSM), which defines bond yields as affine functions of underlying state variables, with affine dynamics. The first general presentation of this class appeared in Duffie and Kan (1996), where the state variables are defined in continuous-time and satisfy a multidimensional diffusion equation with linear drift and volatility. This specification includes as special cases well known single-factor models such as the Vasicek model (Vasicek, 1977), the CIR model (Cox et al., 1985b), and multifactor models such as the two-factor CIR model (Chen and Scott, 1992) or the Longstaff–Schwartz model (Longstaff and Schwartz, 1992). A unified presentation of these basic processes has been developed by Duffie et al. (2003) who consider continuous-time processes with an exponential affine conditional Laplace transform. This extended class of continuous-time affine processes includes affine diffusion processes, as well as jump processes with affine intensity.

However, the flexibility of this class of continuous-time affine models has been questioned in both the applied and theoretical literature. First, several applied studies have rejected different versions of continuous-time affine models (see e.g. Dai and Singleton, 2000; Duffee, 2002). Second, Duffie et al. (2003) mentioned that this class "essentially" includes some mixture of Ornstein–Uhlenbeck, CIR and bifurcation processes and thus seems rather limited. These remarks

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generate an incentive to extend the basic model. Two directions have been followed in the recent literature. The first considers quadratic term structure models (QTSMs), where bond yields are quadratic functions of a multivariate Ornstein–Uhlenbeck process.[1] The second introduces affine processes in discrete time (called compound autoregressive (CAR) processes), defined by the condition (see Darolles et al., 2006):

$$E[\exp u'x_{t+1}|x_t] = \exp[a(u)'x_t + b(u)], \quad \text{for any real } u^2 \tag{1}$$

Since the condition on the Laplace transform is written for the unitary horizon only, instead of being written for any real positive horizon as in the continuous-time approach, we get an infinitely much larger class of affine processes in discrete time than in continuous-time (see Gourieroux et al., 2006).

It has been recently observed that the standard QTSM[2] is a special case of an affine model obtained by stacking the factor values and their squares (Dai and Singleton, 2003b; Cheng and Scaillet, 2007). However, the standard QTSM is still constrained and not able to capture various stylized facts observed on term structure. For instance, it is not convenient for a multivariate modeling of conditional yield volatilities and correlations (Ahn et al., 2003). The limitations of the QTSM may arise from a lack of interpretation of the risk factors introduced in affine models. Indeed, in a structural model, the risk factors can represent the components of preferences (resp. of production activities) related to the risk. These components can be measured by the second-order derivative of a utility function (resp. by the volatility matrix of the activity return), and are naturally represented by symmetric negative (resp. positive) definite matrices. This idea is developed in this paper.

This paper provides an extension of a subclass of QTSM in the framework of the affine class. The extension is based on Wishart risk factors, that are the components of a stochastic symmetric positive definite matrix following a Wishart autoregressive (WAR) process. In this framework, we define the Wishart term structure model (WTSM) and derive simple parameter restrictions that ensure positive bond yields at all maturities. We show that the WTSM is an extension of a subclass of QTSM and can eliminate the degenerate conditional distribution of bond yields arising in QTSM. At first sight the results in this paper seem contradictory with the restrictions on the volatility matrix and state space of an affine process discussed in the literature (see Duffie and Kan, 1996; Dai and Singleton, 2000), requiring that the volatility matrix be diagonal up to a path independent linear invertible transformation. We reconcile our results with previous findings by showing that the standard constraints can be considerably relaxed.

In Section 2 we introduce the WAR process, which is used for specifying the discrete time dynamics of stochastic symmetric positive definite matrices. The Wishart term structure model is developed in Section 3. Section 4 discusses the factors in the standard QTSM and shows that the WTSM is an extension of a subclass of QTSM. In Section 5 we explain why the standard constraints on affine processes mentioned above can be relaxed. Section 6 concludes the paper. The proofs are gathered in Appendices.

2. The Wishart autoregressive process

The Wishart autoregressive (WAR) process is a model for the dynamics of stochastic symmetric positive definite matrices (Y_t) of dimension (n, n) (see Gourieroux et al., 2009, for properties of a WAR process). The distribution of the process (Y_t) is characterized by its conditional Laplace transform, which provides the conditional moments of exponential affine combinations of the elements of Y_t:

$$\Psi_t(\Gamma) = E_t[\exp Tr(\Gamma Y_{t+1})] = E_t\left[\exp\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\gamma_{ij}Y_{ij,t+1}\right)\right], \tag{2}$$

where Tr denotes the trace operator, E_t denotes the expectation conditional on the values Y_τ, $\tau \le t$, and $\Gamma = (\gamma_{ij})$ is a (n, n) real symmetric matrix such that the above expectation exists. The real Laplace transform is defined on a neighborhood of zero and characterizes the distribution due to the positivity of the process (Feller, 1971).

Definition 1. The Wishart autoregressive process of order one, denoted WAR(1), is a matrix process (Y_t) with the conditional Laplace transform:

$$\Psi_t(\Gamma) = \frac{\exp Tr[M'\Gamma(Id-2\Sigma\Gamma)^{-1}MY_t]}{[\det(Id-2\Sigma\Gamma)]^{K/2}}. \tag{3}$$

where K is the scalar degree of freedom, $K > n-1$, M is the (n, n) matrix of autoregressive parameters, Σ is a (n, n) positive definite matrix, and Γ is such that $\|2\Sigma\Gamma\| < 1$, where the norm $\|.\|$ is the maximal eigenvalue.

The WAR process (Y_t) exists for any real degree of freedom such that $K > n-1$. It is a Markov process since the conditional distribution depends on the information set through Y_t only. Moreover, since the conditional Laplace transform is an exponential affine function of Y_t, a WAR process is also a compound autoregressive process, for which conditional moments, and more generally conditional distributions, are easily computed at any horizon (Darolles et al., 2006;

[1] See Longstaff (1989); Beaglehole and Tenney (1991, 1992); Constantinides (1992); Lu (1999); Ahn et al. (2002), Leippold and Wu (2002, 2003), Chen et al. (2004), Cheng and Scaillet (2007).

[2] See Section 4 for a description of the standard QTSM and their properties.

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