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Dynamic portfolio choice under ambiguity and regime switching mean returns

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ABSTRACT

I examine a continuous-time intertemporal consumption and portfolio choice problem under ambiguity, where expected returns of a risky asset follow a hidden Markov chain. Investors with Chen and Epstein's (2002) recursive multiple priors utility possess a set of priors for unobservable investment opportunities. The optimal consumption and portfolio policies are explicitly characterized in terms of the Malliavin derivatives and stochastic integrals. When the model is calibrated to U.S. stock market data, I find that continuous Bayesian revisions under incomplete information generate ambiguity-driven hedging demands that mitigate intertemporal hedging demands. In addition, ambiguity aversion magnifies the importance of hedging demands in the optimal portfolio policies. Out-of-sample experiments demonstrate the economic importance of accounting for ambiguity.

1. Introduction

Since the seminal work of Merton (1971), a number of papers have examined dynamic portfolio choice when investment opportunities are time varying (e.g., Campbell and Viceira, 1999; Kim and Omberg, 1996; Schroder and Skiadas, 1999, 2003). Assuming fully observable investment opportunities, this bulk of literature finds that the intertemporal hedging demand, which arises due to stochastic variation in investment opportunities, is important in portfolio decisions. However, in reality, investment opportunities are partially observable as moments of probability distributions of investment opportunities are often unobservable and must be estimated from observed market signals. Dothan and Feldman (1986) and Detemple (1986) were the first to study asset prices under incomplete information in general equilibrium, followed by David (1997), Veronesi (1999), Ai (2010), among others.[1] Other papers, to name a few, Gennotte (1986), Brennan (1998), Lakner (1998) and Honda (2003), analyze dynamic portfolio choice under incomplete information. Feldman (2007) provides an elaborate review of this literature and related discussions. Recently, Bjök et al. (2010) obtain explicit representations of the optimal wealth and investment processes for a wide range of partially observable investment opportunity sets. This growing body of literature employs recursive-filtering methods to estimate unobservable moments of distributions of asset returns based on observed asset prices. The stochastic processes describing the dynamics of the estimated moments are then treated as perfectly known and optimal consumption and portfolio policies can be derived using techniques for solving complete information economies. All these papers assume that investors have complete confidence in the probability law governing the evolution

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[1] David (1997) investigates unobservable and regime-switching investment opportunities in continuous time. Lundtofte (2008) examines expected life-time utility and hedging demands when endowments and their expected growth rate are imperfectly correlated.

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of the estimated moments with no concerns regarding model uncertainty, which will be relaxed inthis paper. The importance of model uncertainty (or ambiguity) has been largely recognized in both the asset pricing literature (e.g., Anderson et al., 2003; Chen and Epstein, 2002; Epstein and Miao, 2003; Gagliardini et al., 2009; Hansen and Sargent, 2001; Leippold et al., 2008; Trojani and Vanini, 2004) and the portfolio choice literature (e.g., Campanale, forthcoming; Liu, 2010; Maenhout, 2004, 2006; Uppal and Wang, 2003).

My aim in this paper is to examine the effects of ambiguity on intertemporal consumption and portfolio decisions in an incomplete information economy.[2] To this end, I follow Honda (2003) and postulate that expected returns of a risky asset are unobservable and follow a hidden Markov chain. For the sake of analytical convenience, I assume that the hidden Markov chain has two different regimes.[3] Investors update beliefs about the unobservable state according to the Bayes rule. Different from the literature assuming expected utility, I employ Chen and Epstein's (2002) recursive multiple priors utility (hereafter RMPU) to account for ambiguity.[4] The present model, therefore, nests the expected utility model of Honda (2003) as a special case, where there is no ambiguity. Ambiguity and ambiguity aversion are concepts used to describe a decision maker who is uncertain over which probability law describes the dynamics of state variables and is also averse to such uncertainty. In the present model, investors with RMPU endogenously choose the worst-case prior among a prescribed set of different priors inducing different posteriors. Investors take into account not only *incomplete information risk* resulting from time-varying precision of beliefs, but also ambiguity about the probability law governing the dynamics of beliefs.[5] In contrast to the i.i.d. case, the effect of ambiguity varies over time as investors engage in continuous Bayesian revisions under incomplete information. This way of modeling ambiguity builds on the work of Miao (2009), where intertemporal consumption and portfolio decisions are examined without considering specific investment opportunity sets.

I use the Malliavin calculus technique and the Clark–Ocone formula in Ocone and Karatzas (1991) to explicitly characterize the optimal consumption and portfolio policies in terms of the Malliavin derivatives and stochastic integrals. My solutions are based on the martingale method of Cox and Huang (1989).[6] I calibrate the model to historical U.S. stock market data. Numerical calculations of the optimal portfolios are implemented through the Monte Carlo Malliavin derivative (MCMD) method developed by Detemple et al. (2003). Similar to others (e.g., Maenhout, 2004, 2006) I find that ambiguity lowers the total stock demand in all states of the economy. Moreover, under incomplete information, continuous Bayesian revisions interact with time-invariant ambiguity aversion to yield an ambiguity-driven hedging component that is state- and horizon-dependent. This component mitigates the hedging demand for stocks while magnifies the relative importance of hedging demand in the optimal stock demand. Contrary to the well-known advice that investors with long horizons should invest aggressively in stocks, I find that the optimal stock demand is decreasing in the horizon, even when ambiguity is taken into consideration. Ambiguity also affects the economic value of learning about the unobservable state as opposed to assuming i.i.d. returns. Because investors worry that the model driving the dynamics of state beliefs is subject to model uncertainty, the economic value of accounting for regimes and filtering declines under ambiguity. For highly ambiguity-averse investors with very long horizons (e.g., 20 years), there is almost no economic value of accounting for regimes and filtering versus assuming i.i.d. returns.[7] This is in contrast to the previous findings based on expected utility (Xia, 2001; Guidolin and Timmermann, 2007) that portfolio strategies taking into account unobservable states and filtering yield higher utility gains than the i.i.d strategy. Finally, out-of-sample experiments using the CRSP (Center for Research in Security Prices) data from 1996 to 2009 demonstrate that portfolio strategies accounted for ambiguity are superior to those ignoring ambiguity for investors with long horizons.

This paper differs from recent works examining implications of learning under ambiguity (Epstein and Schneider, 2007; Leippold et al., 2008). These papers assume that information on the fundamental process is ambiguous and analyze the belief updating mechanism with multiple priors and likelihoods. Here, investors treat the model driving the dynamics of the filtered probabilities as ambiguous and have multiple beliefs with respect to the Bayesian estimated model. Schroder and Skiadas (2003) examine intertemporal consumption and portfolio policies for generalized recursive utility preferences that incorporate RMPU as a special case. They show that the optimal consumption and portfolio policies can be characterized up to the solution to a single constrained backward stochastic differential equation (BSDE). But they did not consider the role of incomplete information. Sbuelz and Trojani (2008) examine asset prices in a continuous-time exchange equilibrium with locally constrained-entropy RMPU (LCE-RMPU). They exogenously posit that the local bound on the size of ambiguity is some function of time-varying state variables. Without deriving explicit solutions, they identify that the impacts of ambiguity on the optimal portfolio strategy are state-dependent in a non-standard way. Here, I show that even with a constant local bound

[2] Under incomplete information, Cagetti et al. (2002) use the robust control approach and a hidden Markov model to examine asset pricing implications of ambiguity.

[3] Guidolin and Timmermann (2007) analyze asset allocation decisions under multivariate regime switching asset returns. They use a four-regime model to characterize the joint distribution of stock and bond returns. Extending the present paper to the multivariate case would be interesting and is left for the future research.

[4] See Gilboa and Schmeidler (1989) and Epstein and Schneider (2003) for axiomatic foundations for multiple priors utility and recursive multiple priors utility.

[5] I thank David Feldman for suggesting the terminology "incomplete information risk".

[6] Karatzas and Xue (1991) use the martingale method to derive the optimal consumption and portfolio choice under incomplete information. Dybvig et al. (1999) consider the application of the method to time-varying investment opportunities.

[7] I calibrate ambiguity (or ambiguity aversion) based on detection error probabilities. A high level of ambiguity results in a small detection error probability. See Section 3.2 for details.