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ScienceDirect
ELSEVIER Journal of Economic Dynamics & Control 31 (2007) 3478–3502
JOURNAL OF Economic Dynamics & Control
www.elsevier.com/locate/jedc

Pricing of path-dependent American options by Monte Carlo simulation

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Received 11 November 2004; accepted 11 December 2006
Available online 6 February 2007

Abstract

In this paper, we evaluate floating-rate bond options, a variant of path-dependent American options, by Monte Carlo simulation. Assuming that the underlying state variable is Markovian, we show that the price of a floating-rate bond option satisfies a dynamic programming equation. The continuation value in the dynamic programming problem is represented by a conditional expectation. It is shown that the conditional expectation can be transformed to an unconditional expectation, using the Malliavin calculus, which in turn enables us to evaluate the price of the floating-rate bond option by Monte Carlo methods. Some numerical examples are given to demonstrate the usefulness of our method.

JEL classification: C15; C63; G13

Keywords: Malliavin calculus; Dynamic programming; American option; Floating-rate bond

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0165-1889/$ - see front matter
doi:10.1016/j.jedc.2006.12.003

1. Introduction

This paper considers the pricing of floating-rate bond options, a variant of path-dependent American options, by Monte Carlo simulation.

In order to explain floating-rate bond options, consider two time epochs, $T_1 < T_2$ say. The cashflow at time T_2 of a floating-rate bond depends on the value at time T_1 of the underlying state variable and, holders of the option can exercise their rights at any time between T_1 and T_2. Hence, floating-rate bond options are American-type options with the 'look-back' feature.

The pricing of an American-type option can be formulated as a dynamic programming problem after an appropriate discretization in time. Any standard method can be applied to solve the dynamic programming problem backwards in time, starting from the maturity of the option.[1] However, such methods do not work if the option has a path-dependent property, simply because the option price depends on the past and, when calculated backwards, the value of the underlying state variable is not yet determined. This forward feature cannot be handled by the standard dynamic programming approach.

On the other hand, Monte Carlo methods are suitable for the evaluation of path-dependent European options, because sample paths of the underlying state variable can be generated forward in time with ease. Various methods have been proposed to accelerate the speed of convergence in Monte Carlo methods.[2]

Monte Carlo methods cannot be applied to American-type options directly because of the backward feature of the dynamic programming approach. More precisely, in the ordinary American option, the option value in the continuation region (continuation value, for short) can be represented as a conditional expectation of the underlying state variable, $\mathrm{E}[\alpha(X(T))|X(t) = x]$ say, $t < T$. Monte Carlo methods generate sample paths forward in time and, therefore, we must calculate the conditional expectations for all possible $X(t) = x$.

Monte Carlo methods can be applied to American-type options only if the continuation values (or the continuation region) are evaluated efficiently. Recently, various methods have been proposed for this purpose.[3] Among them, the Malliavin calculus transforms the conditional expectations into unconditional expectations, and Monte Carlo methods can be used directly to evaluate the unconditional expectations. See, for example, Fournie et al. (2001), Bally et al. (2005), Bouchard et al. (2004), and Mrad et al. (2003) for applications of the Malliavin calculus to Monte Carlo methods in finance.

The continuation value of a floating-rate bond option can also be represented as a *bivariate* conditional expectation of the underlying state variable, $\mathrm{E}[\alpha(X(T))|X(t_1) = x_1,$

[1]The standard methods include the finite difference method and the tree method. See, for example, Rebonato (1998) for details of such standard methods.

[2]Not only various types of variance reduction methods, but also the use of low-discrepancy sequences can accelerate the speed of convergence. See Jackel (2002) for details.

[3]Such methods include regression-based methods (Longstaff and Schwartz, 2001; Tsitsiklis and Van Roy, 2001), the duality approach (Rogers, 2002), and the stochastic mesh method (Broadie and Glasserman, 2004). See Glasserman (2003) for details.