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E-stability vis-a-vis determinacy results for a broad class of linear rational expectations models ☆

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Abstract

It is argued that learnability/E-stability is a necessary condition for a RE solution to be plausible. A class of linear models considered by Evans, G.W. and Honkapohja, S. [2001. Learning and Expectations in Macroeconomics, Princeton University Press.] is shown to include all models of the form used by King, R.G. and Watson, M.W. [1998. The solution of singular linear difference systems under rational expectations. International Economic Review 39, 1015–1026] and Klein, P. [2000. Using the generalized Schur form to solve a multivariate linear rational expectations model. Journal of Economic Dynamics and Control 24, 1405–1423], which permits any number of lags, leads, and lags of leads. For this broad class it is shown that, if current-period information is available in the learning process, determinacy is a sufficient condition for E-stability. It is not a necessary condition, however; there exist cases with more than one stable solution in which the solution based on the decreasing-modulus ordering of the system's eigenvalues is E-stable. If in such a case the other stable solutions are not E-stable, then the condition of indeterminacy may not be crucial for practical issues.

JEL classification: C62; D84

Keywords: Determinacy; Multiple solutions; E-stability; Learnability

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1. Introduction

Much recent research in economics, especially in monetary economics, has focused on issues involving analytical indeterminacy—multiplicity of stable rational expectations solutions—often in dynamic general equilibrium models based on optimizing behavior by individual agents.[1] In this context, the recent appearance of major publications by Evans and Honkapohja (1999, 2001) has stimulated new interest in the concept of E-stability, developed by DeCanio (1979), Evans (1985, 1986, 1989), and Evans and Honkapohja (1992).[2] The reason is that E-stability is very closely linked with least-squares learnability, and the latter is arguably a necessary property for a rational expectations solution to be plausible as an equilibrium for the model at hand.[3] In their book, Evans and Honkapohja (henceforth, E&H) provide conditions for E-stability of a class of linear multivariate models, but the class in question might appear to be rather restricted in scope. It is shown below, however, that the E&H specification is in fact quite broad, in the sense that essentially any model of the class analyzed by King and Watson (1998) or Klein (2000) can be represented in the implied form. It follows that analytical results shown to hold for the E&H class are actually of quite broad applicability.

In the present paper, consequently, I draw upon results of E&H (1999, 2001) and McCallum (1998) to develop simple proofs of two useful propositions pertaining to this broad class of linear rational expectations (RE) models. The first, Proposition P1, is that if a RE solution is determinate (unique dynamically stable), then it has the property of E-stability (and therefore least squares learnability). The second proposition, P2, is that there exist various cases with a multiplicity of stable[4] solutions in which the one based on the decreasing-modulus ordering of the system's eigenvalues is E-stable. Furthermore, it is a simple matter to determine whether the requisite criteria for E-stability are satisfied.

It should be stated clearly at the outset that all results presented here are based on the assumption that current values of endogenous variables are included in individuals' information sets; if instead only lagged endogenous variables can be observed in the learning process then different E-stability and learnability results would be relevant. Analysis of a few particular problems in monetary economics involving the latter specification has been conducted in a well-known paper by

[1] In monetary economics such issues include indeterminacy under inflation forecast targeting (Woodford, 1994; Bernanke and Woodford, 1997; King, 2000), deflationary traps (Benhabib, et al., 2001), the fiscal theory of the price level (Sims, 1994; Woodford, 1995; Cochrane, 1998; Kotcherlakota and Phelan, 1999; McCallum, 2001), and the validity of the 'Taylor Principle' (Woodford, 2003). For a useful overview of several related points, see Bullard and Mitra (2002).

[2] Evans and Honkapohja (1999) is an extensive survey article in the Taylor-Woodford *Handbook of Macroeconomics*, whereas their (2001) is a major treatise published by Princeton University Press.

[3] This position is developed on pp. 2–3, while Appendix A briefly reviews relevant concepts.

[4] Throughout, the unmodified word 'stable' will refer to the presence or absence of dynamic stability of the rational expectations solution in question, not the learning process or the meta-time concept of E-stability.