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Modeling crack propagation with the extended scaled boundary finite element method based on the level set method

Jian-bo Li [a,*], Xing-an Fu [a], Bai-bin Chen [a,b], Chenglin Wu [c], Gao Lin [a]

[a] Institute of Earthquake Engineering, State Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology, Dalian, Liaoning, 116024, China
[b] China Railway Siyuan Survey and Design Group Co. Ltd., Wuhan, Hubei 430063, China
[c] Research assistant at Dept. of Aerospace Engineering and Engineering Mechanics, University of Texas at Austin, TX 78751, United States

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ABSTRACT

The extended scaled boundary finite element method (X-SBFEM) based on the level set method (LSM) is proposed in this paper to combine the advantages of the scaled boundary finite element method (SBFEM) and the extended finite element method (XFEM). The level set method (LSM) algorithm is applied to further develop the X-SBFEM, especially for the crack propagation problem. The Heaviside enrichment function is used to represent a jump across a discontinuity surface in a split element, and the non-smooth behavior around the crack tip is described using the semi-analytical SBFEM. The stiffness of the region containing the crack tip is computed directly, and the generalized stress intensity factors of many types of singularities are obtained directly from their definitions using consistent formulas. In the numerical simulations, a square plate with an edge crack under tension, a three-point bending beam, a four-point shear beam and a dam (the Koyna dam) with a single propagating crack are modeled. The results show that the proposed X-SBFEM is capable of calculating the stress intensity factors of cracks and predicting crack trajectories and load–displacement relations accurately. An analysis of the sensitivity of the parameters is employed to demonstrate that various mesh densities and crack propagation step lengths led to consistent results.

1. Introduction

In engineering, many catastrophic accidents have occurred because of fracture or crack propagation. Therefore, research into the fracture process of materials is meaningful for improving engineering designs. Many numerical methods, including the finite element method (FEM), the extended finite element method (XFEM) and the scaled boundary finite element method, have been used in studies of crack propagation.

The FEM is very general and flexible in modeling structures with complex boundary conditions and complex cracking patterns [1]. However, complex remeshing is needed when cracks propagate and a 1/4 node singularity element is necessary to reflect the singularity at a crack tip. To obtain sufficiently accurate results, a fine finite element mesh around the crack tip is inevitable.

Belytschko pioneered so-called the extended FEM or XFEM, in which the displacement discontinuities due to cracking are embedded in the finite elements using discontinuous functions and near-tip enrichment functions within the framework of partition of unity so that remeshing is completely avoided [2–4]. The premise for which XFEM can be applied to crack propagation is that the asymptotic field of the crack tip is known in priori in an analytical form and can, therefore, be used to construct a complex enrichment function. For the fracture problem in homogeneous materials, the asymptotic field of the crack tip is complex and unclosed [5]. How to accurately construct enrichment functions that are suitable for numerical calculations needs further study. In addition, complex enrichment functions are added to the finite element method based on the principle of approximation, which adds additional unknown quantities and causes the numerical integration to be difficult at the crack tip and the stiffness matrix to be singular [6].

The scaled boundary finite element method (SBFEM), which was recently developed by Wolf and Song, combines the advantages of the FEM and the BEM. It discretizes boundaries only to reduce the modeled spatial dimensions one, as in the BEM, and does not need fundamental solutions like the FEM does. The result of the radial calculation is completely accurate, and the result of the circumferential calculation is approximately accurate in the

* Corresponding author at: Institute of Earthquake Engineering, The School of Hydraulic Engineering, Dalian University of Technology, No. 2 Linggong Road, Dalian 116024, China. Tel.: +86 411 84708589.

E-mail address: jianboli@dlut.edu.cn (J.-b. Li).

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finite element sense. The SBFEM can improve the computational accuracy significantly [7–9]. It is highly adapted to handling fracture problems and can calculate the stress intensity factors (SIFs) [10] using a unified formula regardless of whether the problem is real, complex, or logarithmic without an integration technique associated with the path of integration. In spite of this, SBFEM can be computationally expensive, may still need to remesh as the crack propagates, and has a full stiffness matrix.

In this paper, for numerical analysis of crack propagation, a newly developed extended SBFEM (X-SBFEM) based on the level set method (LSM) [11,12] is proposed, in which the advantages of the XFEM and SBFEM were combined by making the most of the ability of XFEM to represent a discontinuous displacement field conveniently and the ability of SBFEM to solve stress singularity problems accurately, and the LSM is applied to further develop the X-SBFEM, especially for the crack propagation problem. In chapter 2, we explained the principle of X-SBFEM. In chapter 3, four numerical simulations (a square plate with an edge crack under tension, a three-point bending beam, a four-point shear beam and a dam (the Koyna dam) with a single propagating crack) are modeled to validate the general adaptation of the proposed X-SBFEM. Additionally, some sensitivity analyses demonstrate that various mesh densities and crack propagation step lengths led to consistent results. In chapter 4, we concluded that this paper has developed the X-SBFEM with the LSM to improve the modeling of crack propagation.

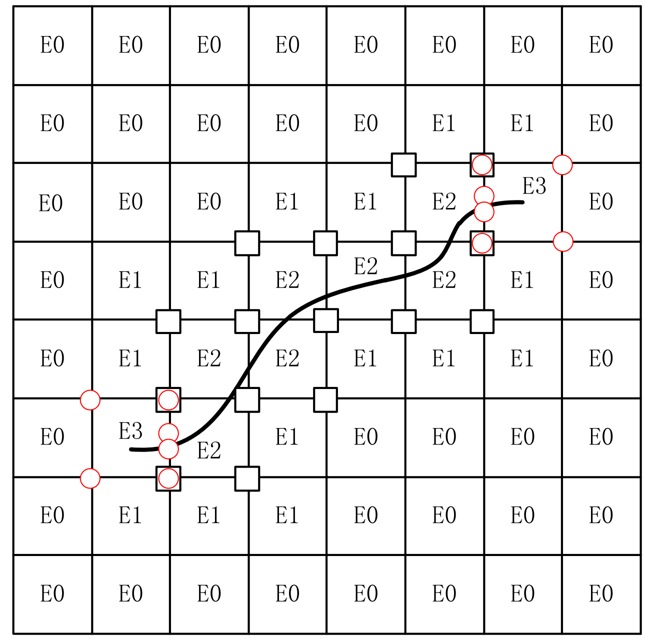

Fig. 1. A typical finite element mesh with an arbitrary crack for the X-SBFEM.

2. The extended scaled boundary finite element method (X-SBFEM)

The core of the X-SBFEM [28,29] is to substitute the semi-analytical SBFEM for the crack tip enrichment function to simulate the nonsmooth behavior around the crack tip while the Heaviside enrichment function is used to represent the jump across the discontinuity surface in the split element. The key is in how the algorithm addresses the boundary conditions at the joint. This method creates four types of elements in the domain: (1) general elements (named E0) with no enriched nodes; (2) mixed elements (named E1) with some enriched nodes; (3) split elements (named E2) with all nodes enriched; and (4) the SBFEM super-element (named E3). Fig. 1 shows a typical finite element mesh and a zone diagram depicting the different element types near an arbitrary crack used in the X-SBFEM. Nodes designated by hollow squares have the number of degrees of freedom of a generalized node, which is used to construct the displacement field in the form of a jump between neighboring elements. Hollow circles are used to designate the nodes that are in SBFEM elements.

2.1. The extended finite element method (XFEM)

The general formula of an XFEM displacement field [13,27] based on linear elastic fracture mechanics (LEFM) is

$$u^h(\boldsymbol{x}) = \sum_{I\in \mathrm{N}^{fem}} N_I(\boldsymbol{x})\boldsymbol{q}_I + \sum_{J\in \mathrm{N}^c} N_J(\boldsymbol{x})\vartheta(\boldsymbol{x})\boldsymbol{a}_J + \sum_{K\in \mathrm{N}^f} N_K(\boldsymbol{x})\sum_{\alpha=1}^{n} B_\alpha(r,\theta)\boldsymbol{b}_K^\alpha \qquad (1)$$

where N^{fem} represents a node in a general element, N^c represents an enriched node in an internal split crack, N^f represents an enriched node that includes the crack tip, and ϑ and B_α represent the discontinuous displacement field at both sides of the crack surface and the enrichment function for the singularity at the crack tip, respectively. $\mathrm{N}_I, \mathrm{N}_J$ and N_K are the shape functions of respective node. $\boldsymbol{q}_I$ is a generalized degree of freedom. $\boldsymbol{a}_J$ and $\boldsymbol{b}_K^\alpha$ are generalized degrees of freedom relating to ϑ and B_α, respectively. n is the number of asymptotic functions of the crack tip, and (r,θ) is the local coordinate of the crack tip.

2.2. The scaled boundary finite element method (SBFEM)

The SBFEM [14,15] uses a radial/circumferential coordinate system. The domain's boundaries are discretized by a one-dimensional line element in the circumferential direction. The local coordinate is $-1 \leqslant \eta \leqslant 1$. A smooth analytic function is used in the radial direction. The local coordinate is $0 \leqslant \xi \leqslant 1$. The scaling center o is selected to ensure that all of the boundaries are visible from it. Specifically, for a fracture problem, the scaling center o is placed at the crack tip.

The displacement field given by the SBFEM is

$$\boldsymbol{U}(\xi,\eta) = \boldsymbol{N}(\eta)\boldsymbol{u}(\xi) \qquad (2)$$

where $\boldsymbol{N}(\eta)$ is the interpolation shape function of the one-dimensional line element.

The governing equations of the SBFEM can be derived using the virtual work principle [16]. Without a body load, the equations are

$$\boldsymbol{E}_0\xi^2\boldsymbol{u}(\xi)_{,\xi\xi} + [\boldsymbol{E}_0 + \boldsymbol{E}_1^T - \boldsymbol{E}_1]\boldsymbol{u}(\xi)_{,\xi} - \boldsymbol{E}_2\boldsymbol{u}(\xi) = \boldsymbol{0} \qquad (3)$$

$$\boldsymbol{P} = \boldsymbol{E}_0\boldsymbol{u}(\xi)_{,\xi} + \boldsymbol{E}_1^T\boldsymbol{u}(\xi)|_{\xi=1}, \qquad (4)$$

where $\boldsymbol{P}$ is the equivalent boundary nodal force, and $\boldsymbol{E}_0, \boldsymbol{E}_1$, and $\boldsymbol{E}_2$ are the coefficient matrices of the SBFEM governing equations.

Because Eq. (3) is the Euler-Cauchy differential equation, its solution must be of the form

$$\boldsymbol{u}(\xi) = \boldsymbol{\phi}\xi^{-\boldsymbol{\lambda}}\boldsymbol{c} = \sum_{i=1}^{n} c_i\xi^{\lambda_i}\phi_i \qquad (5)$$

Substituting Eq. (5) into Eqs. (3) and (4) leads to a standard linear eigenvalue problem [16]. It leads to the modal displacement matrix Φ and eigenvector λ.

On the boundary ($\xi = 1$), Eq. (5) becomes

$$\boldsymbol{u}_b = \boldsymbol{\Phi}\boldsymbol{c} \quad \text{or} \quad \boldsymbol{c} = \boldsymbol{\Phi}^{-1}\boldsymbol{u}_b \qquad (6)$$

Substituting Eqs. (5) and (6) into Eq. (4) leads to

$$\boldsymbol{P} = \boldsymbol{K}\boldsymbol{u}_b = (\boldsymbol{E}_0\boldsymbol{\Phi}\boldsymbol{\lambda}\boldsymbol{\Phi}^{-1} + \boldsymbol{E}_1^T)\boldsymbol{u}_b \qquad (7)$$