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An efficient adaptive frequency sampling scheme for large-scale transient boundary element analysis

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ABSTRACT

The frequency-domain approach (FDA) to transient analysis of the boundary element method, although is appealing for engineering applications, is computationally expensive. This paper proposes a novel adaptive frequency sampling (AFS) algorithm to reduce the computational time of the FDA by effectively reducing the number N_c of sampling frequencies. The AFS starts with a few initial frequencies and automatically determines the subsequent sampling frequencies. It can reduce N_c by more than 2 times while still preserving good accuracy. In a porous solid model with around 0.3 million unknowns, 4 times reduction of N_c and the total computational time is successfully achieved.

1. Introduction

The boundary element method (BEM) has been extensively used to solve elastodynamic problems in many fields of science and engineering, including fracture mechanics [1,2], seismology [3,4] and soil-structure interactions [5]. This paper is devoted to the efficient solution of transient elastodynamic problems.

According to the different solution strategies in time space, the BEM for treating such problems generally follows two approaches, namely, time-domain approaches and frequency-domain approaches; see, e.g., the reviews by Beskos [6] and Costabel [7]. Time-domain approaches can be further classified into time-stepping methods and the space–time integral equation method. In these methods the physical problems are directly solved in the real time domain, thus one can observe the phenomenon as it evolves. However, such methods require an adequate choice of the time step size. An improper time step could lead to instability or numerical damping. For recent development of time-domain methods, see e.g. [8,9]. Beside these methods there exist the possibility to solve the time-domain boundary integral equation with the so-called convolution quadrature method proposed by Lubich [10], which provides a straightforward way to obtain a stable time-stepping scheme using the Laplace transform of the kernel function [8,11,12].

The frequency-domain approach based on the Laplace transforms offers another attractive approach for transient analysis [13–15]. In this approach, one solves the frequency-domain boundary integral equations at a series of discrete frequencies, then obtains the time-domain responses by employing certain numerical inverse Laplace transform methods [16]. The Fourier series method (FSM) [17,18] is one of the most popular methods in computing the inverse Laplace transform which has found wide applications in boundary element transient analysis, see e.g. [19–22]. It works by truncating the infinite Bromwich contour integral of inverse Laplace transform into a finite one, and evaluating the finite integral using the trapezoidal rule based on equally-spaced integration points (i.e., sampling frequencies). As such, the time-domain responses can be efficiently obtained by using the fast Fourier transform [23]; see Section 2.2. In real applications the number of sampling frequencies in the FSM can often be more than one hundred. In large-scale BEM analysis this undoubtedly implies, a quite huge, if not prohibitive, computational burden.

This paper is devoted to the effective reduction of the number of sampling frequencies in the frequency-domain approach. The outcome is an adaptive frequency sampling (AFS) algorithm for transient BEM analysis, in which the sampling frequency is not equally-spaced but adaptively determined according to the characteristics of the computed frequency-domain responses. This work

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is inspired by the fact that the frequency spectrums of most real structures are often smooth apart from certain peaks; and consequently, the equally-spaced sampling frequencies used in the FSM is not optimal in terms of computational efficiency. The adaptive algorithm is built upon the fitting of the frequency response functions using rational functions, which can be efficiently solved by using the vector fitting method proposed in [24]; see Sections 2.3 and 3 for the descriptions of the frequency-domain rational fitting and the adaptive algorithm, respectively.

To the knowledge of the present authors, the present adaptive algorithm is unique in solving transient elastodynamic problems using the frequency-domain approach. It assembles the time-domain responses by using the linear combinations of the frequency-domain responses at some "typical" frequencies; in this sense, it bears a similarity to the method of modal superposition in finite element analysis. The difference is that in the finite element method these "typical" frequencies are the natural frequencies obtained by eigenvalue analysis. However, in BEM we cannot afford to solve large-scale nonlinear eigenvalue problems for the natural frequencies even to date [25]. Our adaptive algorithm works by extracting some "typical" frequencies based on the computed frequency response functions.

Finally, we mention that the idea of reducing the number of solutions in frequency domain in preparation for the numerical inversion is not new. It was first employed by Roesset and Kausel in conjunction with the Fourier transform in [26] and later by Beskos et al. in conjunction with the Laplace transform in [27]. However, in all those works equally-spaced sampling frequencies were used and the number of frequency-domain solutions was reduced by using suitable value of the damping parameter η. This is similar to the FSM. From this point of view, our work builds upon the method used in, e.g., [27]. We achieve further reduction of the frequency-domain solutions by using more efficient, unequally-spaced sampling frequencies, which are adaptively determined according to the actually responses.

Our numerical experiments indicate that, by using the proposed adaptive algorithm instead of the FSM, the number of sampling frequencies can be reduced by a factor of 2 or more, while still preserving the accuracy of the time-domain solutions; see Section 4. This implies considerable saving of the computational time in large-scale transient elastodynamic BEM analysis.

2. Basic theories and methods

2.1. Fast BEM for frequency-domain elastodynamics

Let $\Omega \in \mathbb{R}^3$ denote the region of space occupied by a three-dimensional elastic solid with isotropic constitutive properties defined by Lamé constants λ and μ, Poisson's ratio ν and mass density ρ. The speeds of S and P elastic waves are denoted by $c_s = \sqrt{\mu/\rho}, c_p = \sqrt{\lambda + 2\mu/\rho}$. Assume that the body force vanishes and the initial displacements and velocities are both zero. Then the frequency domain boundary integral equation (BIE) for elastodynamic problem reads

$$c_{ij}(\mathbf{x})u_j(\mathbf{x},s) + (\mathrm{P.V.})\int_\Gamma T_{ij}(\mathbf{x},\mathbf{y},s)u_j(\mathbf{y},s)\,\mathrm{d}\Gamma_{\mathbf{y}} \\ = \int_\Gamma U_{ij}(\mathbf{x},\mathbf{y},s)\sigma_j(\mathbf{y},s)\,\mathrm{d}\Gamma_{\mathbf{y}}, \quad \mathbf{x} \in \Gamma \tag{1}$$

where, s is the complex frequency; u_j and σ_j are components of displacements and tractions in the frequency domain, respectively; (P.V.) indicates a Cauchy principal value (CPV) of the singular integral; the free-term $c_{ij}(\mathbf{x})$ is equal to $0.5\delta_{ij}$ for a smooth boundary at $\mathbf{x}$; $U_{ij}(\mathbf{x},\mathbf{y};s)$ and $T_{ij}(\mathbf{x},\mathbf{y};s)$ denote the displacement and traction fundamental solutions which can be found in many text books and thus are omitted here.

In this paper, the frequency-domain BIE (1) is solved by using the locally-corrected Nyström BEM [28] based on curved quadratic elements. In the numerical implementation, the boundary Γ is partitioned into n_e curved triangular quadratic elements. The 6-point Gauss quadrature rule on triangle is used in evaluating regular element integrals. Thus, the nodes of the Nyström method on each element are the points of the 6-point Gauss rule, and there are totally $n_k = 6 \cdot n_e$ in the boundary element mesh. By adopting the Nyström discretization to all the boundary integrals associated with the components of the kernels U_{ij} and T_{ij} and enforcing the boundary conditions, one can finally obtain a linear system of equations

$$\mathbf{A}(s)\mathbf{a}(s) = \mathbf{b}(s), \tag{2}$$

where, the N by N ($N = 3n_k$) system matrix $\mathbf{A}$ and the N-vector $\mathbf{b}$ are known, and the N-vector $\mathbf{a}$ collects the unknown nodal displacement and traction components which can be obtained by solving the system. Note both the matrix and vectors in (2) are functions of the frequency s. All the nearly singular integrals in the Nyström discretization are evaluated by using the usual recursive subdivision quadrature procedure, and all the weakly and strongly singular integrals are computed by using the method recently proposed in [29]. The linear system (2) is solved iteratively by using the generalized minimal residual method (GMRES). The evaluation of the matrix–vector product is accelerated by using the kernel-independent fast multipole method (KIFMM) [30].

2.2. Frequency-domain approach for transient analysis

In general, by frequency-domain approach (FDA) we mean a method for computing the time-domain responses via the inverse Laplace transform of their frequency-domain counterparts. To be more specific, let $h(s)$ denote a frequency-domain function, which can be the displacement or traction component in BIE (1). Its time-domain counterpart, denoted by $\hat{h}(t)$, can be expressed by using the Bromwich contour integral for inverse Laplace transform

$$\hat{h}(t) = \frac{1}{2\pi i}\int_{\eta - i\infty}^{\eta + i\infty} h(s)e^{st}\,\mathrm{d}s, \tag{3}$$

where, $s = \eta + i\omega$, with ω being the circular frequency; the abscissa of convergence η is a real constant chosen to put the contour to the right of all singularities in $h(s)$, which will be discussed later.

The numerical computation of the Bromwich contour integral (3) is in general an ill-posed problem. This difficulty has led to the diversity of viable numerical approaches in the literature; see, e.g., [18]. In [16] the performance of five different approaches, including the Gaver–Stehfest method, Schapery method, Weeks method, Talbot method and the Fourier series method (FSM), is compared for BEM applications. In large-scale BEM simulations since the solution of linear system (2) is often computationally very expensive, the FSM should be the most economical and robust one in all these five methods. The primary motivation of this paper stems from the promotion of the computational efficiency of the FSM for solving large-scale problems. However, the proposed method in Section 3 is more like a new frequency-domain approach to transient analysis. To inspire the new method, the basic idea of the FSM is briefly summarized.

Let $\Delta\omega$ and Δt be the circular frequency and time resolutions, and T be the time period of the transient response. Given the number of sampling points N_s, one has the basic relations $\Delta\omega = \frac{2\pi}{T}$ and