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Discretization of highly persistent correlated AR(1) shocks

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ABSTRACT

The finite state Markov-chain approximation methods developed by Tauchen (1986) and Tauchen and Hussey (1991) are widely used in economics, finance and econometrics to solve functional equations in which state variables follow autoregressive processes. For highly persistent processes, the methods require a large number of discrete values for the state variables to produce close approximations which leads to an undesirable reduction in computational speed, especially in a multivariate case. This paper proposes an alternative method of discretizing multivariate autoregressive processes. This method can be treated as an extension of Rouwenhorst's (1995) method which, according to our finding, outperforms the existing methods in the scalar case for highly persistent processes. The new method works well as an approximation that is much more robust to the number of discrete values for a wide range of the parameter space.

1. Introduction

The finite state Markov-Chain approximation methods developed by Tauchen (1986) and Tauchen and Hussey (1991) are widely used in economics, finance and econometrics in solving for functional equations where state variables follow autoregressive processes. The methods choose discrete values for the state variables and construct transition probabilities so that the characteristics of the generated process mimic those of the underlying process. The accuracy of the approximation generated by these methods normally depends on the number of discrete values or grids for the state variables, called the fineness of the state spaces, and the persistence of the underlying process. According to Tauchen (1986), Tauchen and Hussey (1991), Zhang (2005) and Flodén (2008), the methods perform poorly for a process whose persistence is close to unity when the state space is moderately refined and hence require a finer state space to achieve a more accurate approximation. However, gaining a closer approximation at the cost of a finer state space may not always work, especially in a multivariate case.

This paper proposes a new method to approximate a particular multivariate autoregressive process, which is referred to as cross-correlated AR(1) shocks. Using appropriate transformations, any vector autoregressive processes can be converted into the process under consideration. The idea behind this method is to decompose the underlying process (carefully while maintaining its basic characteristics) into a set of AR(1) schemes, some of which are independent and the others are perfectly correlated with the independent ones in terms of their error terms. By virtue of the perfectly correlated error terms, the method amounts to constructing transition probabilities for each of the independent AR(1) processes and then generating the other AR(1) processes from the error terms of the independent processes. Using methods that work well in the scalar case, the independent AR(1) processes are accurately approximated. The new method generates accurate

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approximations for a wide range of the parameter space, without requiring a large number of grid points for the state variables.

The independent AR(1) processes under the new method can be approximated by existing methods in the literature for the scalar case. As another contribution of the paper, we compare and contrast the numerical accuracy of these methods. Flodén (2008) examines the performance of the methods of Tauchen (1986), Tauchen and Hussey (1991) and Adda and Cooper (2003). Based on a poor performance of these three methods for highly persistent processes, Flodén modifies Tauchen and Hussey's method and obtains better results for a certain range of the parameter space. In addition to those in Flodén (2008), we include Rouwenhorst's (1995) method in our exercise which considers equispaced discrete values for the state variable and builds the probability transition matrix analytically. The persistence of the process we consider contains values that are sometimes significantly larger than those in Flodén (2008). We find that Rouwenhorst's method outperforms the others for highly persistent processes in the sense that the accuracy of its approximations are robust to the number of grids for the state variable. In general, Tauchen's method tends to overshoot their targets while those of Tauchen and Hussey and Adda and Cooper undershoot when the state space is not sufficiently fine. Moreover, we observe that some of the results in Flodén (2008) are reversed when the process is more persistent than the one he considered. Specifically, as the degree of persistence gets closer to unity, the original version of Tauchen and Hussey's method is able to generate some data which vary over time while Flodén's version of the method cannot.

In the scalar case, more accurate approximations can be achieved without increasing the number of grids for the state variable with all the methods except for Rouwenhorst's. One can use the monotonic relationship between targets and approximations—a one-to-one mapping—in the cases of both overshooting and undershooting. For example, when aiming for the persistence of a process with Tauchen's method, experiment with values smaller than the target and choose the one that yields the closest approximation; or experiment with higher values than the target for the methods that undershoot. However, in the multivariate case, it is difficult to establish the one-to-one mapping between the simulated and targeted parameters as one must experiment with many different coefficients as well as the covariance matrix of the error terms.

The new method can be treated as a multivariate extension of the approximation methods which can work well in the scalar case. Rouwenhorst's method has not been extended to the multivariate case. Therefore, our method can be considered a multivariate extension of Rouwenhorst's method. Another interesting feature of the new method is that instead of applying one method to all the independent AR(1) processes in consideration, one can indeed mix different methods depending on the persistence of the individual processes. For instance, we can use the Tauchen and Hussey (1991) and Rouwenhorst (1995) methods simultaneously (with a moderate-sized state space) by applying the former to the AR(1) processes with sufficiently low degrees of persistence and the latter to highly persistent ones. The rationale of using Tauchen and Hussey's method for low persistent processes is that its approximations of the higher-order moments of the underlying process tend to be slightly more accurate than those of Rouwenhorst's method.

The paper is organized as follows. Section 2.1 shows the shortcoming of the existing methods through Tauchen's method.[1] Section 2.2 discusses the new method and its results in comparison with those in Section 2.1. Section 2.3 demonstrates how to use the new method to approximate VAR(1) processes. Section 3 applies both Tauchen's and the new methods to solve a functional equation of a simplified version of the Mortensen and Pissarides model and compare the results. Finally, Section 4 summarizes the conclusions of the paper.

2. Model

We consider the following multivariate autoregressive process:

$$\begin{aligned} x_{1,t} &= \rho_1 x_{1,t-1} + \varepsilon_{1,t} \\ x_{2,t} &= \rho_2 x_{2,t-1} + \varepsilon_{2,t} \\ &\vdots \\ x_{n,t} &= \rho_n x_{n,t-1} + \varepsilon_{n,t} \end{aligned} \tag{1}$$

where $|\rho_i| < 1$ for all $i \in \{1, 2, \ldots, n\}$, and the innovations, $\varepsilon_t = (\varepsilon_{1,t}, \varepsilon_{2,t}, \ldots, \varepsilon_{n,t})^T$, follow a multivariate normal distribution, $\varepsilon_t \sim \mathrm{N}(0, \Omega)$ with Ω being an $n \times n$ positive definite matrix. It is assumed that ε_t is serially uncorrelated. Given the above specifications, the process in (1) is referred to as cross-correlated AR(1) shocks for the rest of the paper. Using appropriate transformations, any vector autoregressive process can be converted into this process.

Before outlining the new method, we first discuss the disadvantage of the existing methods used in approximating the process in (1). We consider Tauchen's (1986) method as representative as they all perform poorly in the case of highly persistent uncorrelated AR(1) shocks which is a special case of (1).

[1] Considering a different method such as Tauchen and Hussey (1991) or a vector extension of Adda and Cooper (2003) is inconsequential for our purposes as all these methods perform poorly in the case of highly persistent uncorrelated AR(1) shocks, the special case of our multivariate autoregression.