Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER Journal of Economic Dynamics & Control 30 (2006) 1569–1587
JOURNAL OF Economic Dynamics & Control
www.elsevier.com/locate/jedc

The stochastic lake game: A numerical solution

W.D. Dechert[a], S.I. O'Donnell[b,*]

[a] *University of Wisconsin, USA*
[b] *Children's Nutrition Research Center, Baylor College of Medicine, 1100 Bates, Houston, Texas 77030, USA*

Received 8 October 2004; accepted 3 October 2005
Available online 2 May 2006

Abstract

In this paper, we numerically solve a stochastic dynamic programming problem for the solution of a stochastic dynamic game for which there is a potential function. The players select a mean level of control. The state transition dynamics is a function of the current state of the system and a multiplicative noise factor on the control variables of the players. The particular application is for lake water usage. The control variables are the levels of phosphorus discharged (typically by farmers) into the watershed of the lake, and the random shock is the rainfall that washes the phosphorus into the lake. The state of the system is the accumulated level of phosphorus in the lake. The system dynamics are sufficiently nonlinear so that there can be two Nash equilibria. A Skiba-like point can be present in the optimal control solution.

We analyze (numerically) how the dynamics and the Skiba-like point change as the variance of the noise (the rain) increases. The numerical analysis uses a result of Dechert (1978. Optimal control problems from second order difference equations. Journal of Economic Theory 19, 50–63) to construct a potential function for the dynamic game. This greatly reduces the computational burden in finding Nash equilibria solutions for the dynamic game.

JEL classification: C610; C630; C730; Q250; Q530

Keywords: Stochastic optimal control; Markov processes; Stochastic dynamic programming; Lake water pollution; Dynamic games with a potential function

*Corresponding author. Tel.: +1 713 7980305.
E-mail addresses: wdechert@ssc.wisc.edu (W.D. Dechert), odonnell@bcm.tmc.edu (S.I. O'Donnell).
URL: http://algol.ssc.wisc.edu/.

0165-1889/$ - see front matter
doi:10.1016/j.jedc.2005.10.010

1. Introduction

The stochastic dynamic nonlinear model describes the economics of a shallow lake. Lakes provide a variety of economic uses: they are sources of drinking water, fishing, recreational sports and pleasant locations for homes. For agriculture, lakes are used for drainage. Runoff from the fields flows into the streams and rivers that feed the lake.

Much of the agricultural runoff includes phosphorus from fertilizers and animal wastes. Phosphorus is the primary nutrient of algæ and weeds in water. When it grows excessively from an infusion of phosphorus, algæ blooms reduce the oxygen content of the lake and release toxins into the lake. The lake becomes unsafe for certain aquatic species and for recreational purposes.

Recent papers have explored some fundamental issues of the optimal level of pollution in a lake with competing uses (Brock and Starrett, 2003; Carpenter et al., 1999 a,b; Dechert and Brock, 2000; Wagener, 2003; Mäler et al., 2003). Except for a working paper by Ludwig et al. (2002), the research has used the deterministic version of the model. This paper introduces a stochastic version of the lake game.

In the deterministic model, the dynamics converge toward steady states. In the stochastic model, there is a long-run distribution of the state variable whose support is in a region about the steady states. The optimal policy for a set of deterministic models converges to a safe low level of the state variable. In the stochastic version of these models, a run of bad shocks can lead to convergence to a potentially unsafe high level. The risk of such a run, of course, is at the heart of many environmental debates.

Our primary interest in this paper is to solve the stochastic model numerically and to characterize the similarities and differences between the results from the deterministic and stochastic version of the lake game model. We believe that the research used to characterize systems such as shallow lakes can be extended to games describing larger-scale systems, such as coral reefs (McClanahan et al., 2002) and wetlands (Gunderson and Walters, 2002).

In Section 2, we present a stochastic version of the lake model and the dynamics of phosphorus in the lake. In Section 3, we show how the model can be interpreted as an open loop dynamic game. We also show how the stochastic dynamic game can be solved as a single optimal control problem. In Section 4, we discuss the computational methods used to solve the stochastic dynamic programming problem and in Section 5, we discuss the results of the simulations.

2. The stochastic lake model

Mäler et al. (2003) presented a deterministic model in continuous time of the dynamics of phosphorus in a shallow lake. Dechert and Brock (2000) adapted their model to a discrete time model and computed numerical solutions for the optimal policy function. In this section, we introduce shocks to the discrete model.