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Diffusion-induced instability and pattern formation in infinite horizon recursive optimal control

William Brock[a], Anastasios Xepapadeas[b,*,1]

[a]Department of Economics, University of Wisconsin, 1180 Observatory Drive, Madison Wisconsin, USA
[b]Department of International and European Economic Studies,
Athens University of Economics and Business, Greece

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Abstract

This paper develops local stability analysis for deterministic optimal control theory for recursive infinite horizon intertemporal optimization problems where there is a continuum of spatial sites and the state variable can diffuse over these sites. We identify sufficient conditions for a type of local instability which emerges from the interaction of the discount rate on the future, the curvature of the Hamiltonian, and the spatial features of the problem. We call this phenomenon optimal diffusion-induced instability (ODI). We illustrate our analytical methods with three stylized applications. The first application is the optimal management of spatially connected human dominated ecosystems. The second and third applications are harvesting of spatially interconnected renewable resources.
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[*] Corresponding author.
E-mail addresses: wbrock@ssc.wisc.edu (W. Brock), xepapad@aueb.gr (A. Xepapadeas).
[1] Beijer Fellow.

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1. Introduction

This paper develops deterministic optimal control theory for recursive intertemporal optimization problems where there is a continuum of spatial sites and the state variable can diffuse over these sites. We develop a set of sufficient conditions on the Hamiltonian for the optimally controlled system to be locally asymptotically stable. We also develop a set of sufficient conditions on the Hamiltonian for the spatial diffusion to cause local instability of optimal steady states. This contrasts with intuition because we would expect diffusion to contribute to stability, not to instability. We show that if the discount rate on the future is small enough relative to the Hamiltonian curvature (the product of the absolute values of the second derivatives of the Hamiltonian with respect to the state variable and to the co-state variable), the usual value loss turnpike global asymptotic stability result is restored. This is so because if the discount rate on the future is zero, the optimally controlled system minimizes long-run value loss relative to the optimal steady state. The optimal steady state solves a strictly concave optimization problem over a convex set when the discount rate is zero. Intuition suggests that the same type of result should hold if the discount rate on the future is close enough to zero, provided that there is strict Hamiltonian curvature present. This intuition can be made rigorous, as we show below. Optimal diffusion-induced instability (ODI) is more difficult to grasp intuitively. It turns out that diffusion-induced instability in optimal control can only occur when the discount rate is larger than Hamiltonian curvature. If the discount rate exceeds a critical value determined by the Hamiltonian curvature, the relative marginal benefits associated with the optimal control of a state variable in space–time change between a spatially homogeneous and a spatially heterogeneous solution, as the strength of diffusion across sizes increases and the size of the space increases. In this case it may *not* be optimal to have a spatially homogeneous system and ODI emerges as a result of optimizing behavior.

We believe that the new contribution of this paper lies in the development and application of the theory of spatial optimal diffusion-induced stability and instability to infinite horizon recursive optimal control problems of the type that lie at the very foundation of dynamic economic theory. The literature on spatial diffusion-induced stability and instability in dynamical systems is huge, but the literature on optimally controlled spatially connected systems is much smaller. The literature on stability analysis in recursive optimal control problems with infinite dimensional state spaces is even smaller yet (e.g. Carlson et al., 1991). We believe the analysis of ODI developed here is new and identifies a new version of diffusion-induced instability which is different from Turing's (1952) classical case.

In order to develop this theory we first present the Hamiltonian formalism for our spatial context, and then we present our stability analysis. We apply our analysis to two stylized problems of dynamic economics: (i) an optimal ecosystem management model where the ecosystems are spatially connected; and (ii) two renewable resource harvesting models where the renewable resource itself diffuses across space. The final section of the paper presents conclusions and areas for further research.

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