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Pricing derivatives with barriers in a stochastic interest rate environment

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Abstract

This paper develops a general valuation approach to price barrier options when the term structure of interest rates is stochastic. These products' barriers may be constant or stochastic, in particular we examine the case of discounted barriers (at the instantaneous interest rate). So, in practice, we extend Rubinstein and Reiner [1991. Breaking down the barriers. Risk 4(8), 28–35], who give closed-form formulas for pricing barrier options in a Black and Scholes context, to the case of a Vasicek modeling of interest rates. We are therefore in the situation of pricing barrier options semi-explicitly or explicitly (depending on the shape of the barrier) with stochastic Vasicek interest rates. The model is illustrated with a specific contract, an up and out call with rebate, hence a typical barrier option. This example is merely here to show how any standard barrier option can be priced and its Greeks be obtained in such a context. The validity of the approximation is analyzed and the sensitivity to the barrier level and to discretization schemes are also derived.

JEL classification: C60; G10

Keywords: Change of Numéraire; Vasicek model; Barrier option; Markovian approximation

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1. Introduction

In this article, we focus our analysis on the pricing of financial contracts with barriers in a stochastic interest rate environment. The applications of barrier options are multiple and go far beyond the study of derivative products. Barrier options are building blocks of diverse fields such as investment choice theory, the study of the capital structure of the firm (see the standard reference of Black and Cox, 1976 for instance, or the interesting contribution of François and Morellec, 2004), or life insurance (see for instance Grosen and Jørgensen, 2002). Recall that these contracts payoffs depend on whether or not the price of their underlying assets cross a barrier from above or from below. They are the essential part of the standard structured products that are guaranteeing the maximum of a capital and the performance of a financial index.

Barrier options have been studied in great detail for a long time. Under the assumption of a unique and constant interest rate, closed-form solutions were given by Merton (1974) for down and out calls, then by Rubinstein and Reiner (1991) for vanilla barrier options. Other contributions include the works of Geman and Yor (1996) and Pelsser (2000) who priced double barrier options, and the innovative article of Chesney et al. (1997) who introduced Parisian barrier options. The payoff of the latter contracts depends on the time spent above or below the barrier. Later on, Linetsky (1999) pioneered step options. In all these papers, the standard Black and Scholes framework is the starting point and in particular the risk-free interest rate is assumed constant. For short term contracts, a constant term structure of interest rates can be considered reasonable; yet, for medium or long term notes this assumption cannot hold.

Lots of structured products currently traded on the American Stock Exchange involve barrier options and some of them are long term products. Indeed 25.7% of equity-linked notes[1] have their payments driven by a triggered event based on the trajectory of the underlying stock. These equity-linked securities with embedded barrier options represent a total volume of $1,109,518,000. The three bigger issuers in the US are: Wachovia Corporation (Enhanced Yield Securities), Morgan Stanley (HITS: High Income Trigger Securities), Citigroup Funding Inc. (EKLS: Equity Linked Securities). The maturities of these products can be very long in the real market. A lot of barrier equity-linked securities (such as described above) are 1-year contracts. Yet, a reasonable amount of long term barrier index linked notes is currently traded on the American Stock Exchange. Typical medium-index barrier index linked notes are issued by Lehman Brothers, and are linked to the S&P 500 Index, Dow Jones STOXX 50 Index, or Russell 2000 Index (their maturities are ranging between 4 and 5 years). Medium-term notes are also existing and popular products in Europe: we can take for example the famous equity winners, or twin wins, which are basically linear combinations of in and out barrier options with a maturity of 5 years. For these products, it is fully relevant to take into account the

[1]Source: www.amex.com, November 2006.