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Approximating volatility diffusions with CEV-ARCH models ☆

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Abstract

This article develops a new model of the ARCH class which allows volatility to react nonlinearly to past shocks as a function of the past volatility level. We show that this model approximates any CEV-diffusion model for stochastic volatility, and we judge its empirical performance as a diffusion approximation to models of the short-term rate with stochastic volatility and as a filter of the unobserved volatility. We show that the estimation of the continuous time scheme to which the discrete time ARCH model converges can be safely based on simple moment conditions linking the discrete time to the continuous time parameters. A natural substitute of a global specification test for just-identified problems based on indirect inference shows in fact that this approximation to diffusions gives rise to a negligible disaggregation bias. A Monte-Carlo study reveals that the filtering performances of this model are remarkably good, even in the presence of serious misspecification.

JEL classification: C15; E43; G12

Keywords: Stochastic volatility; ARCH filtering; Indirect inference

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1. Introduction

The estimation of continuous time models has recently received increasing attention by both financial economists and macroeconomists. From methods extending the original Hansen's (1982) GMM approach (see, for example, Duffie and Singleton, 1993; Gouriéroux et al., 1993; Gallant and Tauchen, 1996), this literature has evolved towards approximating the maximum likelihood (ML) estimator. Aït-Sahalia (2002, 2003) developed a closed-form approximation to the ML estimator for scalar and multivariate observable diffusions whereby the unknown transitional density of the model can be approximated, in closed-form, with great accuracy. Brandt and Santa-Clara (2002) and Durham and Gallant (2002), both building on Pedersen (1995), proposed to resort to simulating high-frequency paths of the state variables of the continuous time model and by means of these, to recover the unknown transition densities. Empirical results on short-term rate models are concordant, independently of the methodologies employed: traditional univariate diffusions perform poorly relative to bivariate continuous time models in which the interest rate dynamics is coupled with its conditional volatility dynamics.[1] These empirical findings are not unexpected. They simply represent the continuous time counterpart of the universal finding of ARCH-type effects in time series of financial price changes.[2] Their theoretical underpinnings may be found in the initial contribution of Nelson (1990), who showed that the basic ARCH models are reasonable approximations to the typical diffusion processes used by theorists in financial economics.[3] For alternative (non-parametric) approaches to volatility measurements, see Andersen et al. (2002).

In this paper, we study the effectiveness of ARCH-type models as auxiliary devices in a variety of interesting experiments of statistical inference. We pursue two fundamental objectives. We study whether ARCH-type models can be safely used to obtain (i) approximations to diffusions *as well as* (ii) filters in continuous time models with unobservable state variables. Our agenda is to take as given a continuous time data-generating process; and then test whether ARCH-type models can (i) approximate and/or support the estimation of its parameters and (ii) recover the dynamics of the unobservable volatility. This is a research topic started by Nelson (1990) and subsequently, for no apparent reason, abandoned. As an example of the lack of results in this field, Campbell et al. (1997) emphasized in their authoritative textbook that the empirical properties of ARCH as approximations to continuous time stochastic volatility processes "have yet to be explored but will no doubt be the subject of future research" (p. 381). This conjecture was

[1]See, e.g., Andersen and Lund (1997a,b) for results obtained through simulated methods of moments and Durham (2003) for results obtained through simulated ML methods.

[2]The unanimous finding of Arch-type effects in financial data has led researchers (Hull and White, 1987; Wiggins, 1987; Longstaff and Schwartz, 1992; Heston, 1993) to extend early asset pricing theories (e.g., Black and Scholes, 1973; Merton, 1973; Vasicek, 1977) to the case in which volatility evolves in a stochastic manner.

[3]The major contribution of Nelson to this literature can be found in Part II of the book edited by Rossi (1996).