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The Gauss–Seidel–quasi-Newton method: A hybrid algorithm for solving dynamic economic models

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Abstract

The Jacobi matrix required for second-order iterations, such as standard Newton–Raphson procedures, is often too costly to determine for large systems of non-linear equations of a dynamic economic model. In such circumstances, first-order iterative methods are commonly adopted. The problem then, however, is that baseline first-order iterations may not converge. This paper provides a solution to this problem by developing a hybrid method of first- and second-order iterations for solving large-scale dynamic models. The modified algorithm is robust and fast and relative running times increase with the size and complexity of the economic model. As it is easy to implement – only using standard numerical procedures to augment conventional and intuitive first-order iterations – the algorithm is particularly attractive.

JEL classification: C63; C68; E13

Keywords: OLG models; Gauss–Seidel; Newton–Raphson; Quasi-Newton

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1. Introduction

Large-scale heterogeneous agent models are increasingly being used to analyze economic problems. The standard procedures used to solve such models resort to general methods for solving large systems of (non-linear) equations. Three types of conventional solution methods can be distinguished: (i) Newton-based methods such as the L–B–J method (Laffargue, 1990; Boucekkine, 1995; Juillard, 1996; Juillard et al., 1998), (ii) the Fair–Taylor (extended path) method (Fair and Taylor, 1983) and (iii) tatonnement methods (Auerbach and Kotlikoff, 1987), which are used in overlapping generations (OLG) models, in particular.

While baseline L–B–J methods treat general equilibrium models as systems of (non-linear) equations and iterate on this entire system, Fair–Taylor methods break equation systems down into successive time periods. Faust and Tryon (1996) discuss alternatives to this baseline Fair–Taylor approach by considering time blocks rather than single periods with the extreme case being a full time block. An alternative blocking approach often encountered in the literature is the solution of full-equilibrium problems by means of sequences of sub-equilibrium problems. Such schemes are often used to solve multi-country models (Mansur and Whalley, 1982; Van der Laan, 1985; Faust and Tryon, 1996).

Solving successive time blocks is not an appropriate approach in OLG economies because generations overlap between periods. Tatonnement methods provide simultaneous solutions for all time periods, but distinguish between separate *demand* and *supply* models (Auerbach and Kotlikoff, 1987). *Outer loops*, providing solutions for the full equilibrium of the economic model, are performed using block Gauss–Seidel algorithms to update aggregate variables. The latter are inputs to the sub-equilibrium problems solved by separate *inner loop* iterations. As such, the blocking scheme in OLG models is similar to the procedure used in multi-country models referred to above.

The present paper is concerned with, but not restricted to, OLG-type applications where outer loop iterations are used to simultaneously solve all time periods of the model. Conventional Gauss–Seidel (or Gauss–Jacobi) methods update aggregate variables by *first-order* methods. They are slow and may fail to converge. Convergence problems force researchers to rely on *ad hoc* dampening factors that are applied to the aggregate variables for all time periods. Augmenting first-order methods with dampening factors generally helps to increase robustness and speed. These modifications of baseline first-order methods have also been referred to as *fast* Gauss–Seidel (FGS) iterations (Hughes Hallet, 1984). While intuitive, convergence is linear at best and the algorithms may fail to converge at all even after various dampening factors have been tried out.[1]

[1]As an alternative to using *ad hoc* dampening factors, *optimal* dampening factors may be determined or various adaptive techniques may be chosen. These techniques are difficult to implement (Hagemann and Young, 1981; Hughes Hallet, 1982; Judd, 1999). Related modifications of first-order methods are also (successive) over-relaxation methods that dampen or extrapolate equation by equation (Judd, 1999; Ortega and Rheinboldt, 2000).