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A model for cementitious composite materials based on micro-mechanical solutions and damage-contact theory

A.D. Jefferson [a,*], T. Bennett [b]

[a] School of Engineering, Cardiff University, Queen's Building, The Parade, Cardiff CF34 3AA, UK
[b] Department of Civil and Structural Engineering, Sheffield University, UK

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ABSTRACT

A model for simulating micro-cracking in cementitious composite materials is presented. The model employs micro-mechanical solutions of a material with a matrix phase, spherical inclusions and penny-shaped cracks as well as a rough crack contact component. The model follows from that of Reference [Jefferson AD, Bennett T. Micro-mechanical damage and rough crack closure in cementitious composite materials. Int J Numer Anal Methods Geomech 2007;31:133–46], but now a two phase composite is used in place of a homogenous material and the matrix stresses are employed in the evaluation of the micro-cracking. Single point compression and tension strain path examples show that the new formulation improves the accuracy of the model when judged against material data from concrete and mortar specimens.

1. Introduction

The mechanical properties of cementitious composite materials (CCMs), such as concrete and mortar, depend upon the strength and stiffness of randomly distributed coarse and fine aggregate particles, the strength of the bond between the aggregate particles and hardened cement paste (hcp), and upon the amount of moisture in the pores of the material. The behaviour of CCMs is also strongly dependent on the presence and development of micro- and macro-cracks within the material. These cracks may occur within the hcp, in the bond between hcp and aggregate particles and, although less frequently, within the aggregate particles themselves [2].

Detailed knowledge of the structure of these materials and of the mechanisms that lead to their complex behaviour has not led to the development of any one material model that is able to represent their multi-faceted behaviour. However much progress has been made over many years in the development of models which are able to represent some of the macroscopic behavioural characteristics of these materials, including those based on plasticity theory [3–5], damage theories [6,7] and combinations of the two [8,9]. These models often have complex surfaces and evolution equations that require multiple parameters. These parameters can vary significantly from material to material and be difficult to establish yet have a strong influence on behaviour.

An alternative to macroscopic models are models based upon micro-mechanical theories. The expectation in using a micro-mechanical basis for a model is that by modelling the structure and behaviour of the material at the meso (or micro) scale, models can be developed which predict the correct macroscopic behaviour whilst using simple model components and a limited number of physically meaningful parameters. The idea of using models for CCMs based on micro-mechanics is far from new. The development of the micro-plane model, which began over a quarter of a century ago, was inspired by micro-mechanics but development continued on a phenomenological path rather than micro-mechanical mechanistic path [10].

A great deal of work has been carried out during the last few decades on micro-mechanical solutions, much of it directed to the simulation of metal matrix composites. Nemat-Nasser and Hori's text [11] provides details of the major advances up to the date of publication and this includes a full account of the classic work of Budiansky and O'Connell [12].

Much past work has concentrated on deriving elastic moduli for composite materials with inclusions and or voids and cracks. Huang et al. [13] computed effective moduli for elastic materials with spherical inclusions and penny-shaped cracks. Zheng and Du [14] used the effective self-consistent method to derive moduli for composites with multi phases and multiple inclusions and voids. Considerable advances have also been made in recent years on micro-mechanical models for simulating inelastic behaviour [15,16], including a micro-mechanical damage model with cohesive cracks [17].

* Corresponding author. Tel.: +44 (0) 29 20875697; fax: +44 (0) 29 20874597.
E-mail address: jeffersonad@c.ac.uk (A.D. Jefferson).

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There has been less work carried out on the application of micro-mechanical models to cementitious composites than to many other types of composite but some relatively recent work in this area has been undertaken by Pensée et al. [18] and Pensée and Kondo [19], who have developed models based on the solution of an elastic solid containing penny-shaped cracks. Gambarotta [20] developed an anisotropic friction-damage model for this type of material based on the solution of an elastic solid containing a series of plane cracks.

The present authors recently developed a model for CCMs based on micro-mechanical theories in which 'penny-shaped' crack solution was adopted with a local damage evolution model [1]. The model was different from others, in particular, because a new rough crack closure component was integrated with the model. This was shown to naturally produce certain characteristics of the biaxial compressive behaviour of cementitious composite materials which are not simulated by models that do not include the contact component. One of the difficulties with the model however was that evolution was controlled by total strains which resulted in parameters that were not directly related to the strength data generally available for such materials. In an attempt to improve the model, and to include the aggregate particles explicitly, the micro-mechanical solution for a two-phase elastic composite comprising a matrix material with spherical inclusions has been employed in place of that of a homogenous isotropic elastic material. The evolution of micro-cracks is assumed to depend upon the effective matrix stresses and can thus, in principle at least, be related to the actual strength data for the matrix material.

2. Model details

This model is based on a two-component elastic composite comprising a matrix (m) and spherical inclusions (Ω). If viewed as concrete, the matrix material would represent mortar and the inclusions the coarse aggregate particles. Spherical inclusions provide only an idealistic representation of the coarse aggregate but the addition of a second phase with different properties does represent an improvement over the use of an isotropic medium and it allows a direct simulation of the stress differences that occur as a result of the material being heterogeneous. It is further noted that the representation would be closer to reality for a concrete made with rounded gravel aggregate than for one made with angular crushed limestone aggregate.

The volumetric proportion of each of these components is denoted f_m and f_Ω, respectively, with the condition that $f_m + f_\Omega = 1$. Micro-cracks are assumed to develop in the matrix phase and are assumed to depend on both the matrix and average stresses. Furthermore, the micro-cracks are assumed to have rough surfaces which can regain contact with both shear and normal closing strains. The basis of the model is illustrated in Fig. 1,

A Mori–Tanaka averaging scheme [11,21] is used to derive the elastic properties of the two component composite. Expressions for the average strain, average stress, constitutive relationships for the individual components, those for the composite as a whole and an expression relating the matrix to average stress tensors are as follows:

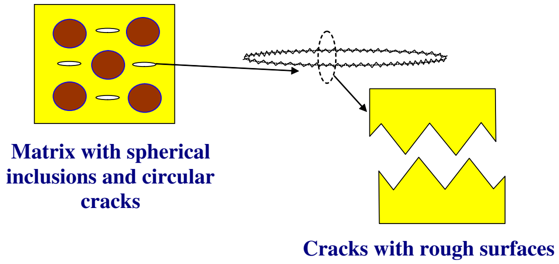

Fig. 1. Two phase elastic composite with elliptical cracks with rough surfaces.

$$\bar{\boldsymbol{\varepsilon}} = f_\Omega \boldsymbol{\varepsilon}_\Omega + f_m \boldsymbol{\varepsilon}_m + \boldsymbol{\varepsilon}_a \tag{1}$$

$$\bar{\boldsymbol{\sigma}} = f_\Omega \boldsymbol{\sigma}_\Omega + f_m \boldsymbol{\sigma}_m \tag{2}$$

$$\boldsymbol{\sigma}_\Omega = \mathbf{D}_\Omega : \boldsymbol{\varepsilon}_\Omega \tag{3}$$

$$\boldsymbol{\sigma}_m = \mathbf{D}_m : \boldsymbol{\varepsilon}_m \tag{4}$$

$$\bar{\boldsymbol{\sigma}} = \mathbf{D}_{m\Omega} : (\bar{\boldsymbol{\varepsilon}} - \boldsymbol{\varepsilon}_a) \tag{5}$$

$$\boldsymbol{\sigma}_m = \mathbf{W}_m \Omega : \bar{\boldsymbol{\sigma}} \tag{6}$$

in which, subscripts m and Ω denote matrix and inclusion phases, respectively, $\boldsymbol{\varepsilon}$ = strain tensor, $\boldsymbol{\sigma}$ = stress tensor, $\mathbf{D}$ = elastic constitutive tensor, $\mathbf{T}_\Omega = (\mathbf{I}^{4s} + \mathbf{S}_\Omega : \mathbf{A}_\Omega)$, $\mathbf{A}_\Omega = [(\mathbf{D}_\Omega - \mathbf{D}_m) : \mathbf{S}_\Omega + \mathbf{D}_m]^{-1} : (\mathbf{D}_m - \mathbf{D}_\Omega)$, $\mathbf{S}_\Omega$ = Eshelby tensor for spherical inclusions, $\mathbf{D}_{m\Omega} = (f_\Omega \mathbf{D}_\Omega : \mathbf{T}_\Omega + f_m \mathbf{D}_m) : (f_\Omega \mathbf{T}_\Omega + f_m \mathbf{I}^{4s})^{-1}$ and $\mathbf{W}_{m\Omega} = \mathbf{D}_m : (f_\Omega \mathbf{D}_\Omega : \mathbf{T}_\Omega + f_m \mathbf{D}_m)^{-1}$, $\mathbf{I}^{4s}$ = symmetric fourth order identity tensor.

The non-zero added strain components from each set of micro-cracks with the same normal vector $\mathbf{r} = [r\ s\ t]^{\mathrm{T}}$ are as follows [11]:

$$\boldsymbol{\varepsilon}_\alpha = \begin{bmatrix} \varepsilon_{\alpha rr} \\ \gamma_{\alpha rs} \\ \gamma_{\alpha rt} \end{bmatrix} = f\,\frac{16(1-\upsilon^2)}{3E} \begin{bmatrix} \bar{\sigma}_{rr} \\ \frac{4}{2-\upsilon}\bar{\sigma}_{rs} \\ \frac{4}{2-\upsilon}\bar{\sigma}_{rt} \end{bmatrix} \tag{7}$$

where $f = N_c a_0^3$, where f is the crack density parameter, N_c is the number of cracks per unit volume and $\bar{\boldsymbol{\sigma}}$ is the far field stress.

If the elastic moduli in Eq. (7) are written in the form of the tensor $\mathbf{C}_\alpha$ and $\bar{\mathbf{s}}$ is the far-field stress tensor transformed to the local axes then

$$\boldsymbol{\varepsilon}_\alpha = f \mathbf{C}_\alpha : \bar{\mathbf{s}} \tag{8}$$

Employing the stress transformation $\bar{\mathbf{s}} = \mathbf{N} : \bar{\boldsymbol{\sigma}}$ and the strain transformation $\boldsymbol{\varepsilon}_\alpha = \mathbf{N}_\varepsilon : \boldsymbol{\varepsilon}_\alpha$, in which $\mathbf{N}$ and $\mathbf{N}_\varepsilon$ denote the stress and strain transformation tensors, respectively, the added strains from a continuously distributed series of cracks is given by

$$\boldsymbol{\varepsilon}_a = \left(\frac{1}{2\pi} \int_{2\pi} \int_{\frac{\pi}{2}} \mathbf{N}_\varepsilon : \mathbf{C}_\alpha : \mathbf{N} f(\theta, \psi) \sin(\psi)\, \mathrm{d}\psi\, \mathrm{d}\theta \right) : \bar{\boldsymbol{\sigma}} \tag{9}$$

in which f is now a continuous function of the 3D polar coordinate angles θ and Ψ.

If the Mori–Tanaka scheme for a non-dilute system was used, the average (far field) stresses ($\bar{\boldsymbol{\sigma}}$ and $\bar{\mathbf{s}}$) would be replaced by the matrix stresses ($\boldsymbol{\sigma}_m$ and $\mathbf{s}_m$). However, if the matrix stresses are used as a basis to calculate the added local strains throughout, the post peak response is particularly brittle in compression. This is because the nature of the homogenised constitutive equation (6) means that lateral matrix stresses do not relax as lateral cracking occurs. For example, under uniaxial compression the lateral zero stress condition is maintained by the lateral tensile matrix stresses balancing the lateral compressive inclusion stresses and these do not both tend to zero directly with lateral cracking but rather maintain the value governed by the compressive stress. Therefore, a cracking stress ($\boldsymbol{\sigma}_\alpha$, $\boldsymbol{s}_\alpha$) is introduced which provides a transition from the matrix to the average stresses, such that initial cracking is controlled by the matrix stresses but the latter stages are controlled by the average stresses as follows:

$$\mathbf{s}_\alpha = \mathbf{N} : \boldsymbol{\Xi} : \bar{\boldsymbol{\sigma}} \tag{10}$$

where $\boldsymbol{\Xi} = r\mathbf{W}_{m\omega} + (1-r)\mathbf{I}^{4s}$, $r = \mathrm{e}^{-\eta_\sigma}$ and $\eta_\sigma = \frac{\zeta_i - \varepsilon_{tm}}{2\varepsilon_{tm}}$ in which ζ is the effective local strain parameter and ε_{tm} the strain at first cracking in the matrix phase. The transition is illustrated in Fig. 2.

Using Eq. (9) in Eq. (5) and making use of Eqs. (4), (6), and (10), the relationship between average stress and average strains may be derived to be