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A test for additive outliers applicable to long-memory time series

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Abstract

We propose a new test for additive outliers in Gaussian time series. The test statistic has a tractable asymptotic null distribution, namely the Gumbel distribution. It is calculated very simply without reference to parameters of any underlying model. The test is valid for a wide class of underlying stationary Gaussian series, and remains valid if the series being tested is pre-filtered by an invertible ARMA filter. To accelerate the convergence to the Gumbel distribution we introduce modified normalization constants and prove their validity. Simulation studies indicate that the test has reasonable power, comparable with a commonly used existing test.

JEL classification: C12; C22

Keywords: Gaussian distribution; Extreme value distributions; Gumbel distribution; Berman condition; Normalization constants

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1. Introduction

Classical statistical analysis of time series is based on the assumption of stationarity and if this assumption is invalid, it is important to become aware of this fact. Departures from stationarity may come about through the time series' having been subjected to exogenous forces or 'interventions', e.g. war, drought, strikes, policy shifts, advertising campaigns and promotions, and pollution catastrophes such as oil-spills.

Methods for dealing with time series data which have been subjected to such exogenous forces constitute 'intervention analysis', an area of study which was first proposed by Box and Tiao (1975), and was extended by Tiao (1985), Chang and Tiao (1983), Chang et al. (1988), Chen and Tiao (1990), Kirkendall (1992), Abraham and Chuang (1993) and Trivez (1995) among others. To a large extent, currently available intervention analysis techniques require that the underlying stationary process have a 'short memory', that is, the process must have a summable autocovariance function. In particular there are substantial difficulties in applying current techniques to fractionally integrated time series.

In intervention analysis a major problem is that of determining whether an intervention has actually occurred. Null distributions for test statistics used in current procedures are unavailable analytically. Hence critical values for such tests can, at best, be produced by means of simulation. Generally speaking it appears that such tests are not actually conducted at specified significance levels. For instance the test procedure proposed by Chang and Tiao (1983), as described in Wei (1990), determines that there is an additive outlier at time T if '$\hat{\lambda}_T = |\hat{\lambda}_{1,T}| > C$, where C is a predetermined positive constant usually taken to be some value between 3 and 4'. The definition of $\hat{\lambda}_T$ (which may be found in Wei, 1990, p. 199) is not important here. The point is that the critical value C is prescribed only vaguely, with no reference to any significance level.

A related issue is that the reliability of currently available tests may be reduced by the fact that pre-estimation of parameters of the underlying series must be performed before the intervention is detected and/or removed. This drawback is especially important with respect to fractionally integrated series for which the estimation of the fractional integration parameter d is a major problem. (See for example Geweke and Porter-Hudak (1983), Yajima (1985), Chen et al. (1994) and Reisen (1994) for a number of procedures for estimating d which have been suggested.)

Box and Tiao (1975) propose a very general intervention model which includes three important types of intervention, namely level shifts, innovation outliers or 'temporary changes', and additive outliers. In this paper we deal only with the last of these three. Assuming that there are (potentially) additive outliers at (usually unknown) times $t_1, t_2, \ldots, t_k$, the model for the time series can be written as

$$Y_t = \mu + \sum_{j=1}^{k} \omega_j a_t^{(t_j)} + X_t, \tag{1}$$