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# Optimality of Ramsey–Euler policy in the stochastic growth model \*

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#### Abstract

For the canonical one sector stochastic optimal growth model, we outline a new set of conditions for a policy function that satisfies the Ramsey–Euler equation to be optimal. An interior Ramsey–Euler policy function is optimal if, and only if, it is continuous or alternatively, if, and only if, both consumption and investment are non-decreasing in output. In particular, we show that under these conditions, the stochastic paths generated by the policy must satisfy the transversality condition; the implication is that in *applying* our result, one does not need to verify the transversality condition when checking for optimality of a policy function.

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#### 1. Introduction

The one sector optimal stochastic growth model (Brock and Mirman, 1972) is the canonical framework used by economists to examine problems of intertemporal resource allocation and more specifically, capital accumulation under uncertainty. It has been widely used as the basic model of macroeconomic growth under technology or productivity shocks and of optimal management of renewable natural resources affected by environmental uncertainty. Variations of the model have also been used to study business cycles.

In this model, a representative agent allocates the currently available output (of a single good) between investment and consumption where consumption generates immediate utility while investment generates next period's output according to a production function that is subject to exogenous production shocks. In the standard version of the model, the exogenous shocks are independent and identically distributed over time. The agent maximizes expected discounted sum of utility from consumption where the discount factor, the utility function and the production function are invariant over time. In such a stationary framework, the intertemporal economic trade-offs faced by the agent are reflected in the *optimal* consumption policy function. This function, which specifies the amount consumed as a function of the current stock of output, has the property that when consumption over time is consistently chosen by following this policy, the path thereby generated is optimal among all paths feasible from the same initial stock. It is important to note that the consumption policy function specifies consumption as a function of current output, regardless of how and when that output level is reached. That is, it does not depend on the date at which the current output is observed, and it does not depend on the history of output levels reached before the current output is observed.

Conditions for optimality play a very important role in understanding the nature of this optimal policy function. In a large class of applications where economists work with specific functional forms for utility and production functions, sufficient conditions for optimality help determine whether an explicitly specified policy function is actually optimal. Even when one cannot derive explicit solutions to the dynamic optimization problem, sufficient conditions for optimality are useful in showing that a certain implicitly defined ("candidate") function is optimal. Necessary conditions for optimality are used to derive qualitative properties of optimal policy functions.

Optimality conditions for the dynamic optimization problem underlying the one sector stochastic growth model can also be useful in dynamic games of capital accumulation such as dynamic games of common property renewable resource extraction.<sup>1</sup>

In a convex framework (strictly concave utility, concave production function), the existing literature has used duality theory to derive a set of conditions that are both necessary and sufficient for a policy function to be optimal and, in fact, to be the unique optimal policy function. In particular, an interior policy function (i.e., one where both consumption and investment are always strictly positive when the current stock of output is strictly positive) is optimal if, and only if, it satisfies the Euler condition (called the Ramsey–Euler equation in this literature) and a transversality condition (Mirman and Zilcha, 1975; Zilcha, 1976, 1978).<sup>2,3</sup>

<sup>&</sup>lt;sup>1</sup> See, for instance, Mitra and Sorger (2014).

<sup>&</sup>lt;sup>2</sup> Key contributions emphasizing the importance of the transversality condition in models of intertemporal resource allocation include Malinvaud (1953), Cass (1965), Shell (1969), Peleg and Ryder (1972) and Weitzman (1973).

<sup>&</sup>lt;sup>3</sup> That the Euler and transversality conditions are necessary and sufficient for optimality has been established for more general, convex dynamic optimization problems. See, among others, Stokey and Lucas (1989), Acemoglu (2009). Estab-

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