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Fairness and efficiency in strategy-proof object allocation mechanisms ☆

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Abstract

I consider the problem of allocating N indivisible objects among N agents according to their preferences when transfers are absent and an outside option may exist. I study the tradeoff between fairness and efficiency in the class of *strategy-proof* mechanisms. The main finding is that for *strategy-proof* mechanisms the following efficiency and fairness criteria are mutually incompatible: (1) *ex-post efficiency* and *envy-freeness*, (2) *ordinal efficiency* and *weak envy-freeness,* and (3) *ordinal efficiency* and *equal division lower bound*. Result 1 is the first impossibility result for this setting that uses *ex-post efficiency*; results 2 and 3 are more practical than similar results in the literature. In addition, for $N = 3$, I give two characterizations of the celebrated random serial dictatorship mechanism: it is the unique *strategy-proof*, *ex-post efficient* mechanism that (4) provides agents that have the same ordinal preferences with assignments not dominated by each other (*weak envy-freeness among equals*), or (5) provides agents that have the same cardinal preferences with assignments of equal expected utility (*symmetry*). These results strengthen the characterization by Bogomolnaia and Moulin (2001); result 5 implies the impossibility result by Zhou (1990).

JEL classification: C78; D71; D78

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1. Introduction

The optimal allocation of goods among individuals is one of the core issues in economics. Normally, researchers analyze this issue using the well-established concepts of markets and auctions, in which individuals receive goods in exchange for transfers. However, in a variety of real-life situations, these transfers are not available for either ethical, institutional or other reasons. Recent literature analyzes numerous examples of such situations. These range from student assignment to primary schools (Abdulkadiroğlu and Sönmez, 2003a) and job placement for graduates (Roth, 1984; Coles et al., 2010), to on-campus housing (Chen and Sönmez, 2002), organ donation (Roth et al., 2004) and distributing military supplies (Kesten and Yazici, 2012).

In this paper I study the simplest version of this class of problems: the object allocation problem,[2] where a set of indivisible objects is allocated to a set of agents solely according to their preferences and such that each agent receives at most one object.[3] The object allocation problem has two stages: first agents report their (ordinal) preferences over objects and then, based on these preferences and using some systematic procedure which we call *a mechanism*, the (probabilistic) assignment is determined. Given the reported preferences and the assignment, we can judge whether the mechanism is efficient (the assignment is not dominated in a certain sense), fair (the agents are treated fairly according to certain criteria), and incentive compatible (agents prefer to report their preferences truthfully). The mutual compatibility of these three types of properties is the focus of this paper.

Since the formal introduction of the object allocation problem by Hylland and Zeckhauser (1979)[4] there has been a search for "nice" mechanisms that would satisfy these major properties: incentive compatibility, efficiency, and fairness. Hylland and Zeckhauser (1979) propose a pseudo-market mechanism that optimally satisfies the latter two properties: the assignment is always *ex-ante efficient* (the assignment is never Pareto dominated) and *envy-free* (each agent prefers her individual assignment to the assignments of others). However, in the pseudo-market mechanism some agents can benefit by misreporting and therefore the mechanism is not *strategy-proof*. Because of this room for profitable manipulation one cannot tell whether the outcome is fair and efficient under the true preferences.

The further search for "nice" mechanisms that are *strategy-proof* gave rise to a series of negative results. Gale (1987) was the first to conjecture that for an object allocation problem with at least three agents, no mechanism can satisfy *ex-ante efficiency*, *strategy-proofness*, and *anonymity*. (*Anonymity* requires that if any two agents exchange the reports, then their assignments are also exchanged.) Later, Zhou (1990) showed a stronger result, where instead of *anonymity* he used *symmetry*. (*Symmetry* requires that any two agents with identical reported cardinal preferences get the same expected utility; it is implied by *anonymity*).

[2] The object allocation problem is also known as the assignment problem and the house allocation problem. Occasionally, I will refer to objects as to houses.

[3] Each agent might receive an outside option.

[4] Hylland and Zeckhauser considered cardinal input, in this paper I mostly focus on ordinal input, but also incorporate few cardinal axioms.