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Stationary Markov perfect equilibria in discounted stochastic games [☆]

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Abstract

The existence of stationary Markov perfect equilibria in stochastic games is shown under a general condition called "(decomposable) coarser transition kernels". This result covers various earlier existence results on correlated equilibria, noisy stochastic games, stochastic games with finite actions and state-independent transitions, and stochastic games with mixtures of constant transition kernels as special cases. A remarkably simple proof is provided via establishing a new connection between stochastic games and conditional expectations of correspondences. New applications of stochastic games are presented as illustrative examples, including stochastic games with endogenous shocks and a stochastic dynamic oligopoly model.

JEL classification: C72; C73

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1. Introduction

The class of stochastic games enriches the model of repeated games by allowing the stage games to vary with some publicly observable states. It has considerably widened the applications of repeated games.[1] In particular, a stochastic game is played in discrete time by a finite set of players and the past history is observable by all the players. At the beginning of each stage, Nature draws some state randomly. After the state is realized, the players choose actions and each player receives a stage payoff that depends on the current state and the actions. The game then moves to the next stage and a new random state is drawn, whose distribution depends on the previous state and chosen actions. The procedure is repeated at the new state. Each player's total payoff is the discounted sum of the stage payoffs.

A strategy for a player in a stochastic game is a complete plan of actions, which specifies a feasible action for the player in every contingency in which the player might be called on to act. However, the so-called Markov strategies, which only depend on the current state instead of the entire past history of states and action profiles, have received much attention in the literature. As noted in Maskin and Tirole (2001), the concept of Markov perfect equilibrium, which requires the players to use only Markov strategies, embodies various practical virtues and philosophical considerations, including conceptual and computational simplicity. Given that the relevant parameters in a stochastic game are time-independent, it is natural to require the Markov strategies to be time-independent as well. Equilibria based on such strategies are called stationary Markov perfect equilibria. In a stationary Markov perfect equilibrium, any subgames with the same current states will be played exactly in the same way. So "bygones" are really "bygones"; i.e., the past history does not matter at all.

Beginning with Shapley (1953), the existence of stationary Markov perfect equilibria in discounted stochastic games remains an important problem. Existence results on such equilibria in stochastic games with compact action spaces and finite/countable state spaces have been established in a number of early papers.[2] Given the wide applications of stochastic games with general state spaces in various areas of economics, the existence of equilibria in stationary strategies for such games has been extensively studied in the last two decades. However, no general existence result, except for several special classes of stochastic games, has been obtained in the literature so far.[3]

[1] See, for example, the book Neyman and Sorin (2003) (in particular the survey chapter Amir, 2003 on economic applications), and the recent survey chapters Jaśkiewicz and Nowak (2016a, 2016b).

[2] The result for zero-sum games with finite actions and states was established in the seminal paper by Shapley (1953). The case with finite state spaces and compact action spaces was shown in Fink (1964) and Takahashi (1964). The result in Shapley (1953) was extended by Parthasarathy (1973) to nonzero-sum games with finite action spaces and countable state spaces. The existence result for countable state spaces and compact action spaces was then proved by Federgruen (1978). Approximate equilibrium has been considered in Whitt (1980) and Nowak (1985). For more detailed discussions about the literature on stochastic games, see the surveys Jaśkiewicz and Nowak (2016a, 2016b).

[3] Stochastic games possessing strategic complementarities were studied in Curtat (1996), Amir (2002), Nowak (2007), Vives (2009), Balbus et al. (2014) and Jaśkiewicz and Nowak (2015); see Section 6 in the survey Vives (2005) for further discussions. A related class of stochastic games in which the interaction between different players is sufficiently weak was studied in Horst (2005). The existence of stationary p-equilibria for two-player games with finite actions and