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The structure of Nash equilibria in Poisson games

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Abstract

We show that many results on the structure and stability of equilibria in finite games extend to Poisson games. In particular, the set of Nash equilibria of a Poisson game consists of finitely many connected components and at least one of them contains a stable set (De Sinopoli et al., 2014). In a similar vein, we prove that the number of Nash equilibria in Poisson voting games under plurality, negative plurality, and (when there are at most three candidates) approval rule, as well as in Poisson coordination games, is generically finite. As in finite games, these results are obtained exploiting the geometric structure of the set of Nash equilibria which, in the case of Poisson games, is shown to be semianalytic.

JEL classification: C70; C72

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1. Introduction

Games with population uncertainty and, in particular, Poisson games (Myerson, 1998), have been proposed to model economic scenarios where it is more reasonable to assume that agents only have probabilistic information about the number of players. Poisson games have been primarily used to study voting games (for a very incomplete list, see Myerson, 2002; Bouton and Castanheira, 2012; Bouton and Gratton, 2015; Bouton, 2013; Gratton, 2014; Huges, 2016) but they also have been proven useful in more general economic environments

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where the number of economic agents is uncertain (see, e.g., Satterthwaite and Shneyerov, 2007; Makris, 2008, 2009; Ritzberger, 2009; McLennan, 2011; Jehiel and Lamy, 2014). Similarly to finite games, Poisson games can generate multiple equilibrium outcomes and, moreover, some of them may be only generated by equilibria that are not a plausible description of rational behavior. The economic literature has typically addressed the issue of multiplicity of equilibrium outcomes investigating conditions that guarantee they are at least locally unique (Arrow and Hahn, 1971, Chapter 9). On the other hand, the literature on equilibrium refinements has helped reducing the indeterminacy of game-theoretical models by eliminating implausible equilibria. In this paper, we expand the analysis of the determinacy of equilibria to Poisson games.

Local uniqueness of equilibrium outcomes in a theoretical model is a basic requirement from an applied standpoint. Otherwise, the theory would be too ambiguous in its description of what to expect from the economic agents. Furthermore, without such a property when performing comparative statics, small variations in the environment can lead to abrupt changes in behavior and in the ensuing outcomes. Local uniqueness of equilibrium outcomes is equivalent to finiteness if the strategy space is compact, which is typically the case. Debreu (1970) argues that even if it is too demanding a requirement that a model has finitely many equilibrium outcomes for *every* possible description of the environment, in some cases, it is possible to show that "most" of them satisfy such a finiteness result. Harsanyi (1973) initiated the analysis of the determinacy of equilibrium in game theory and proved that, for generic assignments of utilities to strategy profiles, every normal form game has finitely many equilibria. This result is of limited significance because in many game-theoretical models (such as voting games) many different strategy profiles lead to the same outcome and, therefore, lead to non-generic games in the space of normal form payoffs. Henceforth, the literature has provided analogous results for extensive-form games (Kreps and Wilson, 1982), sender–receiver games (Park, 1997), voting games (De Sinopoli, 2001), network formation games (Pimienta, 2009) and other families of finite games (Govindan and McLennan, 2001). We show how many of these results can be extended to Poisson games. In particular, Poisson voting games under plurality, negative plurality, and approval rules (the latter when the number of candidates is no larger than three), as well as Poisson coordination games, have finitely many Nash equilibria for generic utilities over the relevant outcome space.

Generic determinacy results also have a practical implication for strategic stability, which has proved a powerful tool to characterize rational behavior in economic models. In finite games, the set of Nash equilibria consists of finitely many connected components. This is a key result towards a satisfactory theory of equilibrium refinements that conceives a solution to a game as a set-valued object (Kohlberg and Mertens, 1986; Mertens, 1989). If there are finitely many equilibrium outcomes then every point in an equilibrium component necessarily induces the same outcome. In these cases, a set-valued solution concept contained in a connected component of equilibria constitutes a minor departure from a classical single-valued one, as all the points in the same connected component can be considered equivalent. Motivated by this, in Section 3 we prove that every Poisson game has finitely many Nash equilibrium components and that at least one of them contains a stable set.

Stable sets in Poisson games are introduced in De Sinopoli et al. (2014) where it is shown that they satisfy existence, admissibility, and robustness against elimination of dominated actions and inferior replies. A stable set in a Poisson game is defined as a set of equilibria that is robust against a suitably chosen family of perturbations. A perturbation in this family is not a strategy tremble as in Kohlberg and Mertens (1986), but rather, consists of pushing with vanishing probability the population's average behavior towards a completely mixed measure on the probability simplex over the set of actions in the game. De Sinopoli et al. (2014) show that