Available online at www.sciencedirect.com

ScienceDirect

Journal of Economic Theory 169 (2017) 425–452

JOURNAL OF Economic Theory

www.elsevier.com/locate/jet

On the set of extreme core allocations for minimal cost spanning tree problems

Christian Trudeau [a,*], Juan Vidal-Puga [b]

[a] *Department of Economics, University of Windsor, 401 Sunset Avenue, Windsor, Ontario, Canada*
[b] *ECOSOT (Economics, Society and Territory) and Departamento de Estatistica e IO, Universidade de Vigo, 36200 Vigo (Pontevedra), Spain*

Received 30 November 2015; final version received 14 November 2016; accepted 3 March 2017

Abstract

Minimal cost spanning tree problems connect agents efficiently to a source when agents are located at different points and the cost of using an edge is fixed. We propose a method, based on the concept of marginal games, to generate all extreme points of the corresponding core. We show that three of the most famous solutions to share the cost of mcst problems, the Bird, folk and cycle-complete solutions, are closely related to our method.

JEL classification: C71; D63

Keywords: Minimal cost spanning tree problems; Extreme core allocations; Reduced game; Bird solution; Folk solution; Cycle-complete solution

1. Introduction

Minimal cost spanning tree (mcst) problems model a situation in which agents are located at different points and need to be connected to a source in order to obtain a good or information. Agents do not care if they are connected directly to the source or indirectly through other agents. The cost to build a link between two agents or an agent and the source is a fixed number, meaning

* Corresponding author.
E-mail addresses: trudeauc@uwindsor.ca (C. Trudeau), vidalpuga@uvigo.es (J. Vidal-Puga).

http://dx.doi.org/10.1016/j.jet.2017.03.001

that the cost is the same whether one or ten agents use that particular link. Mcst problems can be used to model various real-life problems, from telephone and cable TV to water supply networks.

The core of mcst problems has been an early focus of attention, with Bird (1976) and Granot and Huberman (1981) showing that it is always non-empty and Granot and Huberman (1984) providing an algorithm to generate multiple core allocations. For the special case for which all edges have a cost of 0 or 1 (called elementary cost matrices in this paper), Kuipers (1993) shows that all extreme points of the core are marginal vectors (a property labeled as the CoMa property by Hamers et al., 2002). One can thus find all extreme points of the core by enumerating all marginal vectors and verifying which ones belong to the core. We present an improvement over these results by providing a method that allows to obtain the full set of extreme core allocations for all cost matrices.

The method is based on the concept of marginal games (Núñez and Rafels, 1998), in which we assign an agent her marginal cost to join the grand coalition, remove her from the problem and update the stand-alone costs of the remaining coalitions: they can either keep their original stand-alone cost or the stand-alone cost of them with the departing player, net of her cost share. This reduction is itself a special case of the Davis–Maschler reduction (Davis and Maschler, 1965). Given an ordering of the agents, repeating the process until all players are removed allows to find an extreme core allocation.

This method or very similar ones have been implemented for the assignment problem (Núñez and Rafels, 2003) and shortest path problems (Bahel and Trudeau, 2014), among others.[1] In the non-cooperative setting, a similar approach consists in ordering buyers according to a given permutation and letting them buy goods in that order (Pérez-Castrillo and Sotomayor, 2002; Vidal-Puga, 2004).

The method does not work as well on all problems. Even though there exist sufficient conditions for the method to always generate the full set of extreme core allocations (Potters et al., 1989; Driessen, 1988; Núñez and Rafels, 1998), none of them are satisfied, in general, by mcst problems. We are still able to prove that the method generates the full set of extreme core allocations, using a representation of marginal games as minimal cost spanning tree problems with priced nodes. This new problem is a generalization of both mcst problems and Steiner tree problems (Hwang and Richards, 1992; Skorin-Kapov, 1995).

By taking the average of these extreme core allocations for all permutations, we obtain a very natural cost sharing solution, identified as the selective value in Vidal-Puga (2004) and that coincides with the so-called Average lexicographic value, or Alexia (Tijs, 2005). If the game is concave, it also corresponds to the Shapley value. We show that our procedure is very close to three well-known cost-sharing solutions for mcst problems.

Firstly, if we only consider permutations that correspond to the order in which we connect agents in an optimal network configuration, we obtain directly the Bird solution (Bird, 1976). The Bird solution was the first solution to be shown to always be in the core and it is known for its simplicity, as we may assign cost at the same time as we construct an optimal tree.

Secondly, we show that for elementary problems (where all costs are either 0 or 1), our solution corresponds to the cycle-complete solution (Trudeau, 2012). The cycle-complete solution is obtained by modifying the cost of some links before taking the Shapley value of the corresponding cost game: we reduce the cost of edge (i, j) if there exists a cycle that goes through

[1] A related approach is that of Tijs (2005); Funaki et al. (2007); Kongo et al. (2010); Tijs et al. (2011), who also look for extreme core allocations given some lexicographic order. However, their approach is explicitly based on the core constraints and not on marginal games.